A hierarchical Varying Sampling $\mathcal{H}_\infty$ Control of an AUV ⋆

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Abstract: This paper deals with the robust control of an Autonomous Underwater Vehicle (AUV) subject to real-time constraints. A new hierarchical structure is presented, considering the strong link between the altitude and the pitch angle. Two different controllers have to be computed, both using the $\mathcal{H}_\infty$ framework. Moreover the measurement of the altitude by an ultrasonic sensor induces a non periodicity in the altitude control. To take into account this variation of the sampling interval, an LPV polytopic controller is designed whose matrices are scheduled w.r.t this parameter. Moreover the proposed design ensures a performance adaptation when the sampling interval varies. This methodology is developed for AUV, which are a difficult to control using linear tools due to strong non linearities.

Keywords: AUV, Robust control, Gain scheduling, Varying Sampling.

1. INTRODUCTION

Autonomous Underwater Vehicles (AUV) are increasingly used for ocean survey, mapping, and data sampling. Various kind of underwater activities and constraints has lead to the development of different kind of vehicles, e.g. slender bodies or open-frame bodies, equipped with different kind of actuators and various set of sensors. The control of underwater vehicles is made difficult by numerous non-linearities, due to cross-coupled dynamics and hydrodynamic forces subject to large uncertainties. Therefore many diverse controllers and control architectures for AUVs have been proposed in the past years. Among others let us cite decoupling steering, diving, and speed control by PID (Jalving (1994)), or coupled PID and anti-windup control (Miyamaoto et al. (2001)) which are based on the linear framework. Sliding mode control (Healey and Lienard (1993)) or $\mathcal{H}_\infty$ control (Feng and Allen (2004)) are ways to handle uncertain parameters and to enforce the robustness. In Roche et al. (2009), Linear Parameter Varying (LPV) polytopic framework has been used to adapt the controller to a variation in the mass of the vehicle. In Silvestre and Pascoal (2004) gain-scheduling is used to adapt the control parameters w.r.t. the variations of the vehicle's forward velocity. Accurate modeling motion induced forces interacting with the vehicle lead to develop model-based observers and controllers as in Refsnes et al. (2008).

Most of these controllers have been designed in the framework of continuous time systems despite they must be implemented using an embedded computing system. Often the controllers of non-linear systems are assumed to be sampled fast enough to make the induced discretization disturbance vanish. However this is not always possible, in particular for embedded systems where the computing power is strongly limited. In that case it is well known that a dynamic management of the control interval is an effective way to control the computing power needed for control (Robert et al. (2010)). Another sampling constraint may come from the acoustic sensors often used in underwater activities. They are characterized by a low bandwidth and a slow propagation time, so that the distance to a target can be measured only with a varying and distance-dependent significant delay. Moreover cross-talking between sensors in a given area must be avoided, e.g. using Time Division Multiple Access (TDMA) to schedule the communications inside a swarm of AUVs (Marques et al. (2007)). In that particular case the communication pattern is pre-defined so that the instant of the next available measurement can be known.

In this paper a new hierarchical control structure for the vertical behavior of AUVs is presented, taking into account the strong link existing between the altitude and the pitch angle. The control of the pitch angle will be achieved using the $\mathcal{H}_\infty$ framework. At the upper level, to control the altitude, a varying sampling rate is considered, assuming that the next sampling instant is known (either estimated or controlled). Moreover, only the bottom following mode is presented in this paper.

The next section describes the AUV non-linear model and its linearization for controller synthesis. In section 3 the design of a pitch angle controller using $\mathcal{H}_\infty$ framework

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is presented. Then an altitude controller scheduling w.r.t the sampling interval is built using the LPV/$\mathcal{H}_\infty$ control design. The last section presents some simulation results and some future perspectives are drawn.

2. AUV MODELS

The vehicle considered here is the *Aster^x* AUV designed and operated by IFREMER (Figure 1). The model is adapted from Fossen (1994), and described in more details in Roche et al. (2010).

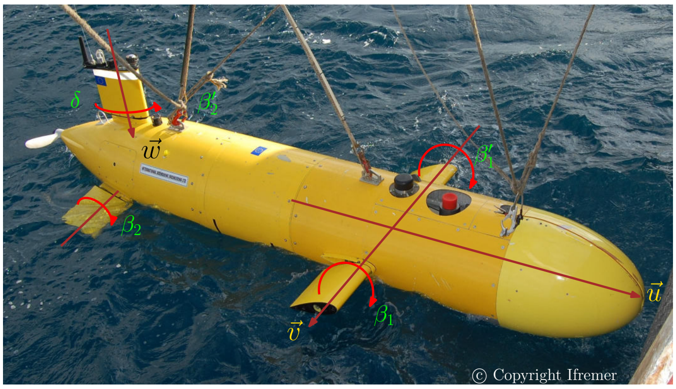

Fig. 1. Frames and actuators

For the description of the vehicle behavior, we consider a 12 dimensional state vector : $X = [\eta(6)\ \nu(6)]^T$.

$\eta(6)$ is the position, in the inertial referential $\mathcal{R}_0$, describing the linear position η_1 and the angular position η_2: $\eta = [\eta_1\ \eta_2]^T$ with $\eta_1 = [x\ y\ z]^T$ and $\eta(2) = [\phi\ \theta\ \psi]^T$ where x, y and z are the positions of the vehicle , and ϕ, θ and ψ are respectively the roll, pitch and yaw angles.

$\nu(6)$ represents the velocity vector, in the local referential $\mathcal{R}$ (linked to the vehicle) describing the linear and angular velocities (first derivative of the position, considering the referential transform) : $\nu = [\nu_1\ \nu_2]^T$ with $\nu_1 = [u\ v\ w]^T$ and $\nu_2 = [p\ q\ r]^T$

The AUV is actuated using 6 inputs:

- a forward force Q_c for the axial propeller
- 2 horizontal fins in the front part of the vehicle (controller with angle β_1 and β_1')
- 3 fins at the tail , one vertical (angle δ) and 2 tilted with angle $\pm\pi/3$ (controlled by angle β_2 and β_2')

2.1 Model linearization

The nonlinear model includes 12 state variables and 6 control inputs. For the computation of the controller, a linear model is proposed. The equilibrium point is chosen as $[u\ v\ w\ p\ q\ r] = [1\ 0\ 0\ 0\ 0\ 0]$: all velocities are taken equal to 0, except the longitudinal velocity taken equal to 1m/s, the cruising speed chosen by the operator according to the payload requirements.

Tangential linearization around the chosen equilibrium point yields to a model of the form :

$$\begin{cases} \dot{X} = AX(t) + BU(t) \\ Y = CX(t) + DU(t) \end{cases}$$

where

- X stand for the state : $X = [x\ u\ y\ v\ z\ w\ \phi\ p\ \theta\ q\ \psi\ r]^T$
- U for the control input $U = [\beta_1\ \beta_1'\ \beta_2\ \beta_2'\ \delta_1\ Q_c]^T$
- Y for the measured output (here only the altitude z is measured)

2.2 Model Reduction

The complete control of the vehicle is intricate due to the large size of the system. A usual solution is to separate the whole model into three different sub-models with reduced size. This allows for a "decoupling" control synthesis for the horizontal and vertical plans.

To control the altitude z, the model is reduced to 4 state variables : z, θ and the corresponding velocity w and q. For the actuation, only 4 fins are needed: the 2 horizontals fins in the front part of the vehicle (β_1 and β_1') and the 2 tilted fins at the tail (β_2 and β_2'). Since the AUV has to stay in the vertical plan, both pairs of fins have to be actuated in the same way (with the same angle) so the control variables are chosen such as: $\beta_1 = \beta_1'$ and $\beta_2 = \beta_2'$.

Remark : In this paper we focus on the control of the altitude z with adaptation to the sampling period w.r.t. the measurement time, following the bottom referenced altitude control scenario. The forward speed u is also controlled using basic (i.e. constant sampling) feedbacks to keep forward velocity constant (more details can be found in Roche et al. (2010)).

2.3 Depth cascade control structure

Assuming a forward velocity fast enough to provide lift forces, the motions in the vertical plane are controlled by the pairs of front and rear control surfaces through their angles β_1 and β_2. Thanks to these two separate actuators, a torque around the pitch axis and a thrust along the z vehicle axis can be theoretically generated independently, and motions along the vertical axis and around the pitch axis could be decoupled, e.g. allowing for vertical motions while keeping the vehicle body horizontal.

However this kind of trajectory generates a lot of drag due to the incident angle between the hull and the fluid. As onboard energy storage is a crucial and limited resource for an AUV, incident angles between the vehicle's body and the forward trajectory must be limited as far as possible. In consequence the lift efforts due to the front and rear tilted fins must be coordinated to keep the vehicle tangent to its trajectory.

Obviously the AUV's altitude is strongly related to the pitch angle, and the best way to climb a slope is to keep the AUV parallel to the slope by first controlling this pitch angle (Varrier (2010)).

Therefore a hierarchical control structure is considered here, as in Figure 2.

The altitude controller K_z (that will be scheduled by the sampling interval) gives a reference of pitch angle, and this reference is used by the pitch angle controller K_θ (constant sampling) to compute the actions to apply to the AUV.

Note that the maximum slope that can be tracked by the AUV depends on the combination between its hydrostatic

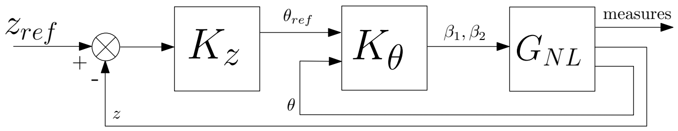

Fig. 2. Cascade control configuration for altitude control

stability, where the return torque is a function of the metacentric distance, and the maximum lift forces on the control surface which increases with the square of the forward velocity. Therefore the input of the altitude controller, which is primitively built from the observed distance to bottom, must be filtered to request only feasible trajectories.

Similar cascade structures has been already described, e.g. in Leveille (2007) where the PD/PI cascaded control structure is assumed to provide a good disturbances rejection. In this paper the $\mathcal{H}_\infty$ framework will be used for the controller computation, since it can ensure performances of the closed loop system and also robustness w.r.t uncertainties, and can also be extended to LPV case (to build gain-scheduled controller).

3. $\mathcal{H}_\infty$ PITCH ANGLE CONTROLLER

This part states the problem in a similar way to (Skogestad and Postlethwaite, 2005).

3.1 The $\mathcal{H}_\infty$ pitch angle control design

This paragraph details the $\mathcal{H}_\infty$ control design for the pitch angle. The first step is to choose a control structure and some weighting functions representing the performance specifications (settling time in closed loop, tracking error, robustness margin...). Usually, a simple mixed-sensitivity algorithm is sufficient to handle the closed-loop performance specifications, while satisfying the actuator constraints. This problem, which is referred to as the S/KS consists in solving:

$$\min \gamma \text{ s.t.} \left\| \begin{matrix} W_e S \\ W_u K_\theta S \end{matrix} \right\|_\infty \leq \gamma$$

where $S = 1/(1 + G.K_\theta)$

In this way the control structure is as follows (figure 3) where :

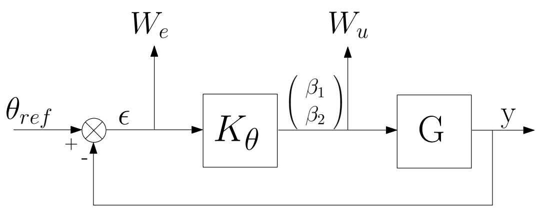

Fig. 3. Structure chosen for the control design

- W_e weights the tracking error, with:

$$\frac{1}{W_{e_\theta}} = \frac{s + w_{b_e}\epsilon}{\frac{s}{M_{se}} + w_{b_e}} \tag{1}$$

  Here the parameters have been chosen to establish some performances of the closed loop system :
  - · a good robustness margin (module margin of 0.5)
  - · a tracking error less than 10^{-4}
  - · a settling time around 1 seconds
- W_u weights the control input $U = \binom{\beta_1}{\beta_2}$, chosen as:

$$\frac{1}{W_{u_i}} = \frac{s + w_{b_u}\epsilon}{\frac{s}{M_{su}} + w_{b_u}}, \; i = \beta_1,\, \beta_2$$

  The parameters in W_u take into account some limits in the gain of the actuators and allows to obtain a good rejection of noise measurement. Since the pitch angle is controlled using 2 control inputs, W_u is chosen as a diagonal transfer matrix, where each component is dedicated to a single actuator ($W_{u_{\beta_1,\beta_2}}$).

After the discretisation of the generalized plant (pitch reduced model and weighting functions) at the sampling period $T_e = 0.01sec$, the solution of the discrete-time $\mathcal{H}_\infty$ optimal control problem (using the *dhinflmi* function of Matlab©) leads to:

$$\gamma_{opt} = 5.4$$

It has been obtained a 7^{th} discrete-time order controller (sum of the order of the reduced model + the orders of the weighting functions). The obtained sensitivity function is shown below.

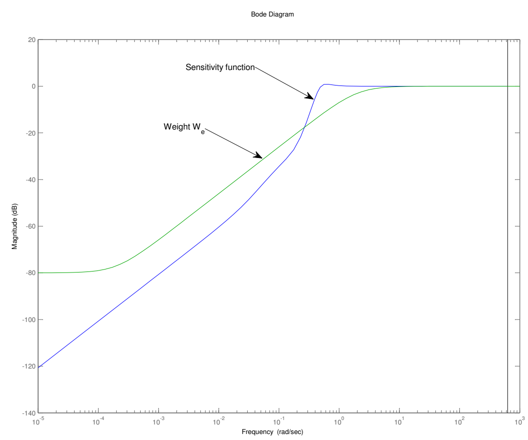

Fig. 4. Sensitivity functions of the pitch control

The bandwidth of the sensitivity function is $0.4rad/sec$: this is a little slower than the one chosen for the weighting function on the tracking error W_{e_θ} (this explain the value of $\gamma > 1$).

4. LPV/$\mathcal{H}_\infty$ CONTROL OF THE ALTITUDE

In Roche et al. (2010), an LFT (Linear Fractional Representation)approach has been used to build a LPV model of the AUV by considering the sampling interval as varying parameter. Here, the polytopic approach will be used to take into account the sampling variation in the model formulation (as in Robert et al. (2010)).

4.1 A control-oriented model for the altitude control:

The geometrical relation between the altitude and the pitch angle will be used to obtain a simple model for the altitude control design(see Figure 5).

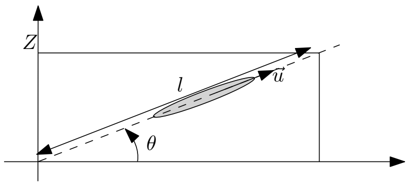

Fig. 5. Relation z θ

$$z = l \sin\theta$$

i.e.

$$\dot{z} = \dot{l}\sin\theta + l\dot{\theta}\cos\theta$$

Moreover, the longitudinal speed u is equal to $\dot{l}$:

$$\dot{z} = u\sin\theta + l\dot{\theta}\cos\theta \tag{2}$$

A 1^{st} order limited development of equation (2) for $\dot{\theta} = 0$ and θ in the neighborhood of 0 implies :

$$\begin{aligned} \dot{z} &\simeq u\theta \\ G_z &= \frac{z}{\theta} = \frac{1}{p} \end{aligned} \tag{3}$$

Therefore, the inner loop composed by the non linear model and the pitch controller G_z can be approximated by an integrator. In this case, the obtained controller will be of low order.

4.2 Discrete time model with varying sampling period

As emphasized in the introduction, the objective is to handle asynchronous measurements in the control algorithm. The design of sampling dependent controller has already been considered by the authors in Robert et al. (2010), giving, in particular, a methodology to obtain a polytopic discrete-time model where the variation of the sampling interval (around the nominal period) is taken into account.

Here, as shown above, the control-oriented model is a simple integrator, making the discretization step much simpler. Indeed the model given in (3) can be converted in discrete-time as follows:

$$z_{k+1} = z_k + h.\theta_k \tag{4}$$

The sampling period is assumed to belong to the interval $[h_{min}, h_{max}]$ with $h_{min} > 0$. It is then considered around the nominal value h_0 as :

$$h = h_0 + \delta \qquad \text{with} \qquad h_{min} - h_0 \leq \delta \leq h_{max} - h_0 \tag{5}$$

Then the 'altitude' discrete-time model is :

$$z_{k+1} = z_k + (h_0 + \delta).\theta_k \tag{6}$$

As the obtained model is affine in the varying parameter δ, and δ is bounded (due to the boundedness of the sampling interval $h \in [h_{min}, h_{max}]$, this model is polytopic.

4.3 LPV polytopic altitude controller

As explained before, this controller will be gain scheduled with respect to the sampling interval. This will be achieved using LPV/$\mathcal{H}_\infty$ framework. To ask for some performance specifications in the closed loop behavior, the structure presented in figure 3 is still used, with new weighting functions.

- $W_e(h)$ is a weight on the tracking error, for fixing specifications on the controlled outputs y (here only the altitude z has to be controlled). The varying sampling period considered for the controller synthesis imposes a parametrized discretisation of weights. This allows the adaptation of the performances with respect to the current sampling period, as explained in Robert et al. (2010). So $W_e(h)$ is defined as the following state space system, with the frequency $f = 1/h$:

$$\begin{cases} \dot{x} = (a \times f)x + (a \times f - b \times f)u \\ y = x + u \end{cases} \tag{7}$$

  This weight is then discretized to obtain discrete-time representation :

$$W_d(z) : \begin{cases} x_{k+1} = A_d\, x_k + B_d\, u_k \\ y_k = x_k + u_k \end{cases} \tag{8}$$

$$\begin{cases} A_d = e^{af\,h} = e^a \\ B_d = (af)^{-1}(A_d - I)bf = a^{-1}(A_d - I)b \end{cases} \tag{9}$$

  The simplification between h and f leads to a discrete LTI (Linear Time invariant) representation of the weight.

  The elements a, and b are chosen to obtain :
  · a good robustness margin
  · a tracking error less that 1%
  · a settling time of about 5 seconds
- W_u is chosen to account for actuator limitations and is taken constant $W_u = 0.02$.

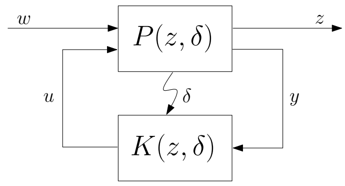

Fig. 6. General control configuration

Then the augmented plant ($P(z, \delta)$ in Figure 6) is built merging the model of the system and the weighting functions.

Finally the LPV polytopic controller is obtained following the methodology presented in Robert (2007) (adaptation of the discrete-time controller synthesis developed in Apkarian and Gahinet (1994) and Scherer and Wieland (2004) to polytopic systems).

4.4 Controller computation results

Using the LPV model for the altitude, and considering a sampling period varying inside the interval $[h_{min}, h_{max}] = [0.05, 0.3]s$ (this corresponds to an altitude between 75 and 450 meters, considering the speed of ultrasound in water) the $\mathcal{H}_\infty$ optimal control problem leads to $\gamma = 0.5$. The following figures give the S sensitivity function and the Bode Diagram of the controller, considering 10 frozen values of sampling period inside the interval.

The S sensitivity function shows the adaptation of the settling time (linked to the the bandwidth) with respect to the current sampling period. The discrete-time weight

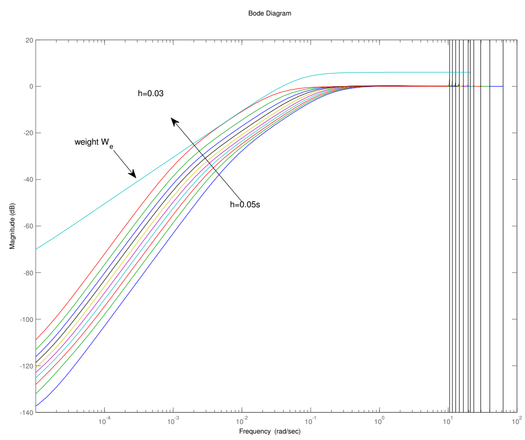

Fig. 7. S sensitivity function

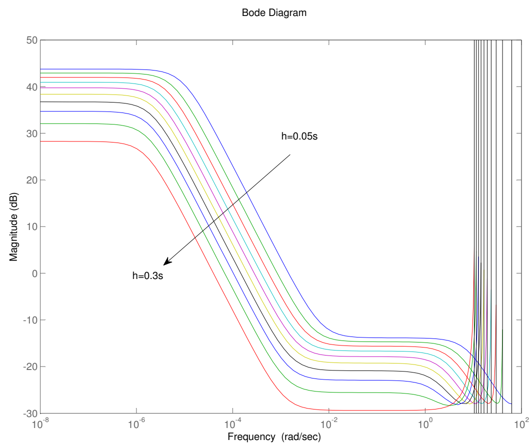

Fig. 8. Bode Diagram of the LPV polytopic controller

$W_e(h)$ is plotted on the same figure, showing the respect of this design specification.

The controller Bode Diagram (Figure 8) shows the adaptability of the gain and the bandwidth to the sampling interval variation.

5. SIMULATION RESULTS

5.1 Simulation scenario

The complete non-linear model of the AUV is used for the following simulations.

The mission considered is to follow the sea bottom by keeping the forward speed constant (this is required for a good interpretation of the results to build a cartography of the seabed).

Both obtained controllers in previous sections are included in the control scheme, considering the chosen structure (see Figure 2).

An independent discrete-time controller controls the cruising speed u which starts at 0 and stays constant and equals $1m/s$ during all the simulation (its design will not be detailed here; it is a simple $\mathcal{H}_\infty$ discrete-time controller, with a sampling period of 0.1s).

5.2 Results

During the simulation, some changes in the sampling interval will show the adaptability of the controller. Here a sinusoidal variation between $0.05s$ and $0.3s$ on the sampling interval is presented. A second order reference on the altitude is applied at minimal and maximal value of the sampling period and h changes (in a sinusoidal way) during the tracking of the altitude. The LPV altitude controller computes a pitch angle reference while the second controller (pitch) uses this value to actuate the non-linear system.

Results are presented on figures 9, 10 and 11 :

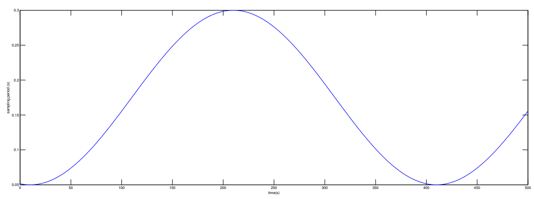

Fig. 9. Sampling period

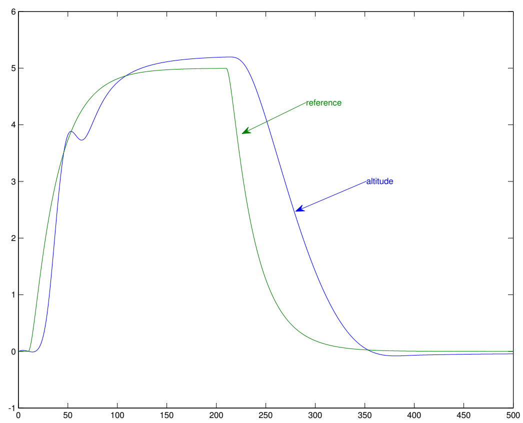

Fig. 10. Altitude

When the sampling period is minimal, at the beginning of the simulation, the tracking of the altitude is achieved despite some small oscillations, due to oscillation on the pitch angle. Indeed it has been applied a controller synthesized w.r.t. the linearized model to the non-linear system; for small control intervals and high bandwidth requirements we approach the capabilities of the non-linear systems and the bounds of the domain where the linearized model is valid. When the sampling interval is maximal, the performances obtained are very satisfactory for the pitch angle, but induce a transient tracking error for the altitude. This shows the adaptation of performances w.r.t the sampling interval : when the value of h is too high, performances are deteriorated.

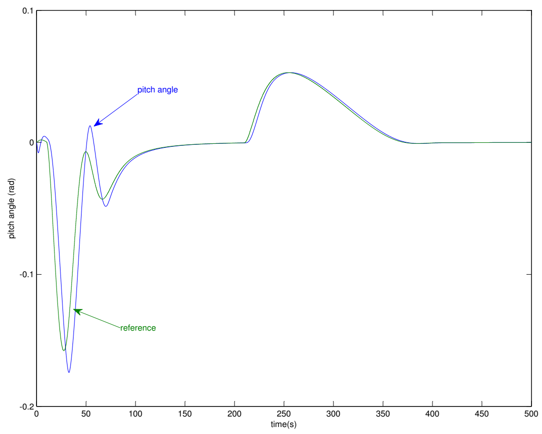

Fig. 11. Pitch angle

It is worth noting that this performance adaptation (deterioration) preserves the closed-loop stability while fixed performance specifications might have led to instability when the sampling rate varies, as explained in Robert et al. (2010).

These simulation results using the non-linear model of the vehicle show the interest of the method : thanks to the robustness brought by $\mathcal{H}_\infty$ methodology, the controllers designed for linear model work on the non-linear one. Moreover, even if the range of variation of h (altitude controller) is important ($h_{max} = 6 \times h_{min}$) the system remains stable : the LPV controller computation ensures the stability whatever the variations (and speed of variation) inside the interval $[h_{min}, h_{max}]$ are.

The results presented here could be improved in term of settling time or tracking error, but are of interest since they give good results even on the non-linear model.

6. CONCLUSION

In this paper a new hierarchical control structure has been presented, to consider the strong link between altitude and pitch angle. An LTI controller has been computed for the pitch angle, and an LPV polytopic one was built for the altitude, both using the $\mathcal{H}_\infty$ framework. The control of the altitude has been scheduled according to the sampling interval to handle asynchronous measurements.

In the work presented here, a single varying parameter has been considered : the sampling interval. But other parameters could be added, such as the forward speed, which is one of the parameters that induces large nonlinearities in the model, and is also important during missions in the sea.

Controllers have been computed based on a linear model, and then successfully applied to the non-linear model in simulations, showing some robustness property. Some uncertainties could also be added for the robust stability analysis of the obtained controller.

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