# Robust control under weakened real-time constraints

Patrick Jocelyn Andrianiaina, Alexandre Seuret and Daniel Simon

***Abstract*— A weakened implementation scheme for real-time feedback controllers is proposed to reduce the conservatism due to traditional worst-cases considerations, while preserving the stability and control performance. Based on recent results to assess stability of linear systems with delayed and sampled-data inputs, this paper takes into account both the effects of deadline misses of control tasks and uncertainties in the plant. The methodology is applied to the pitch control of an aircraft, showing that weakening the real-time constraints allows for saving computing power while preserving the system's stability.**

***Index Terms*— Robustness, delay, real-time scheduling.**

## I. Context

The development process of critical avionics products are done under strict safety regulations. These regulations include determinism and predictability of the systems' timing. The overall approach is based on a separation of concerns between control design and implementation, [1], [2].

On one hand, traditional control design considers constant sampling rates with equidistant samples (e.g. no jitter) and negligible, or fixed and known delays. On the other hand real-time scheduling theory has mainly focused on how to dimension resources for meeting deadlines (or equivalently, on the schedulability analysis for a given resource) [3]. Therefore the computer science and real-time scheduling communities do their best to implement control tasks considering fixed periods and hard deadlines, and assuming that the Worst-Case Execution Time (WCET) is precisely known. This assumption has served the separation between control and scheduling designs, but leads to an under utilization of CPU resources, and such approach faces both technical, economical, and industrial challenges.

One of the toughest challenges in the current approach is the determination of the WCET, in order to correctly size the system. The tightness of the result is related to the predictability of the processing unit. The future generations of processors seems to go apart from the predictability and determinism objectives of the execution time. Processing speeds and performances grow up very fast thanks to accelerating but unpredictable mechanisms of new processors. However it becomes difficult to foresee their effects on the execution time considered in the worst case. Nowadays, even if many attempts are proposed to give an upper bound of the WCET (e.g. [4]), both the traditional and current approaches are difficult to be applied to modern processor generations and produce values which are pessimistic [5].

Then, to implement the control laws the hard and costly way consists in building a highly deterministic system so that the actual implementation parameters meet the ideal ones. By essence, implementations purely based on WCET and hard deadlines considerations are conservative and lead to a large under-utilization of the computing and networking resources and finally ending to an oversizing of both electrical supplies, cooling systems and aircraft weight.

Current real-time systems design methods and associated analysis tools do not provide a model flexible enough to fit well with control systems requirements, while classic control theory does not give advice on how to include resource and dependability constraints into the controller, both at the design and implementation stage. However, as far as *closed-loop* control systems are considered, more flexible solutions can be expected by exploiting the basic features of feedback loops, robustness w.r.t. modeling uncertainties, disturbance rejection and adaptiveness to various operative conditions. Indeed robustness of feedback controllers also implies some fault-tolerance w.r.t. deviations from the ideal timing pattern, e.g. equidistant sampling. This feature can be efficiently used to guarantee the end-to-end control quality, i.e. stability and performance level, under weakened real-time constraints, therefore improving the computing power average utilization.

The remainder of the paper is organized as follows: Section II provides the motivation of the paper and the problem statement. This section also gives different points of view on design assumptions concerning critical system design. Section III formulates the problem of systems under uncertainties and input delays. Section IV provides new stability conditions, based on [6], for systems subject to uncertainties, delays and varying sampling. Finally in section V the methodology is applied to the case study of the pitch control of an F-16 aircraft.

**Notations:** Throughout the article, the sets $\mathbb{R}^+$, $\mathbb{R}^{n\times n}$ and $\mathbb{S}^n$ denote respectively the set of positive scalar, the set of $n \times n$ matrices and the set of symmetric matrices of $\mathbb{R}^{n\times n}$. The superscript '$T$' stands for the matrix transposition. The notation $P > 0$ for $P \in \mathbb{S}^n$ means that $P$ is positive definite. For any matrix $A \in \mathbb{R}^{n\times n}$, the notation $2He\{A\} > 0$ refers to $A + A^T > 0$.

## II. Problem statement and paper contribution

### A. WCET based assumption

Currently many control systems, e.g. flight control, braking control systems, are considered to be hard real-time,

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i.e. it is most often assumed at design time that control tasks must be executed strictly periodically. Therefore control tasks executions are bounded to fixed time-slots, and deadline misses or jitter are forbidden. It is assumed that any deviation from the ideal timing pattern inevitably leads to a failure of the system. The implementation of such control tasks relies on a safe evaluation of the WCET of each task, which is used to dimension the size of the time slot allocated for the execution of the control tasks. The execution schedule of a control task is depicted on Figure 1.

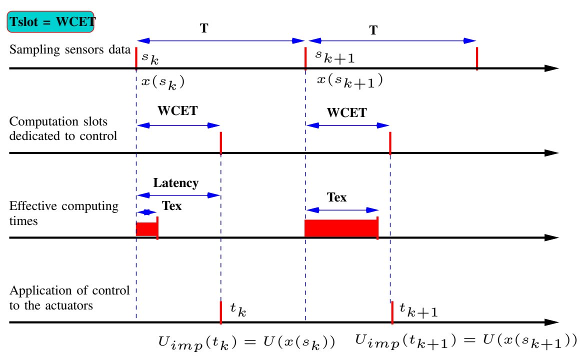


Fig. 1. WCET based control task execution pattern

A given task execution is strictly periodic, i.e. a time slot $T_{slot} = WCET$ is allocated to the task execution. It is triggered at a period $s_k - s_{k-1} = T$ by the occurrence of measurements $x(s_k)$ at time $s_k$. The controller computation takes a time $T_{ex}$ which is always smaller than the $WCET$. To avoid jitter, the control signal $U(x(s_k))$ is applied to the actuators at the end of the slot, i.e. at time $s_k + WCET$:

$$\forall t \in [s_k + WCET, s_{k+1} + WCET[, \quad U = U(x(s_k)).$$

Therefore, it is a periodic control system, with constant period $T$, subject to a constant delay $T_{slot} = WCET$. This implementation fits with the hard real-time assumption, and should be applied when the controller is really hard, e.g. if it is a Finite State Machine which may fail in an unpredicted state in case of deadline miss and interrupted transition.

However, as the time slots are allocated based on the WCET of the control tasks, the computations always finish before the end of the slot. Therefore, a fraction of the computing power is unused. The wasted computing power is all the more important as the WCET is far from the average value of the observed execution time $T_{ex}$. Indeed, due to an increasing demand on services, new control systems are more and more based on networked architectures and shared off-the-shelf computing devices. However high computing power are often based on the usage of multiple levels of cache and pipe-lines, lowering the determinism of the processors and increasing the difficulty of searching for the program's WCET, which are, in fact, approached by increasingly conservative upper bounds [7]. Indeed the execution times distribution plot (as shown in Figure 2 for a dedicated embedded processor [8]) is expected to spread out, so that approaching the worst case execution time is foreseen to become a rare event. Therefore the amount of wasted computing power is expected to increase, leading to costly over-sizing of embedded computers, power supplies and cooling systems.

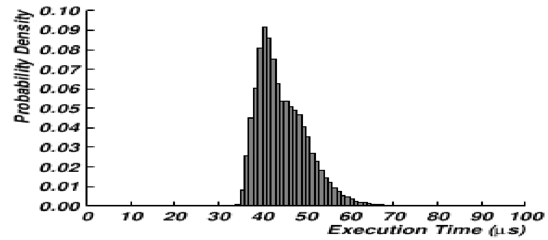


Fig. 2. Typical execution time distribution

Hence it is worth to discuss and revisit the "hard real-time" assumption and examine how it can be weakened in the particular case of *feedback* control systems.

### B. Robustness considerations

The design of critical systems must satisfy requirements, specifications and certification levels. Robustness is a general concern that grows with system complexity. For instance, it is known that small task core execution time modifications in systems with complex performance dependencies can have drastic non-intuitive effects on the overall system performance, and might lead to constraint violations. [9] claims that robustness evaluation using simulation is a tedious tasks and practically impossible for the reason that simulation models do not support many of the possible property changes (for instance, increased processor execution times or modified communication volumes). This same paper proposes an interesting formal approach to robustness of embedded real-time systems with definition of robustness metrics. However the present article uses recent developments and moreover is more dedicated to practical industrial problems.

Robustness in control usually deals with the plant's parameters uncertainties, but in the present case the insensitivity or adaptability w.r.t. timing deviations from the theoretical pattern, such as jitter or deadlines misses, is also investigated. For SISO linear systems robustness can be quantified using phase margins, delay margins and module margins. It appears that a phase margin implies a delay margin (i.e. the maximum unmodeled constant extra delay that can be suffered before instability) and certainly a jitter margin, which is more difficult to quantify ([10]) but which can be experimentally shown ([11]). A feedback control system can be even robust enough to tolerate missed samples, e.g. in [12], where selective data dropping is applied to lighten the computing and networking burden while preserving closed-loop stability. The interesting point is that a feedback control system which is robust w.r.t. the plants parameters uncertainties is also robust, to some extent, w.r.t. timing deviations. Hence a feedback control system is not as hard as it is often considered in the literature, but should be better considered as *weakly hard*, i.e. able to tolerate specified timing deviations without leaving its requested performance [13].

### C. Weakened real-time and control objectives

To improve the average efficiency of embedded computers while preserving the control stability and performance, and relying on the robustness of feedback control laws, it is proposed to weaken the usual real-time constraints according to the following schedule and control objectives (Figure 3):

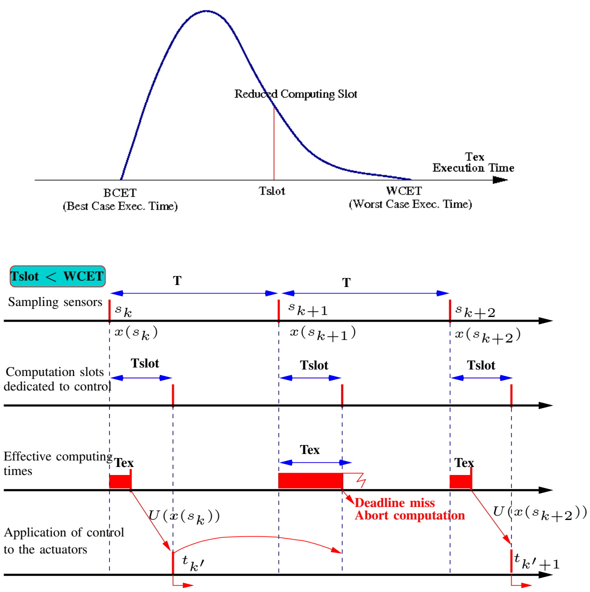


Fig. 3. Proposed task execution pattern

The sensors data are still expected to occur at a fixed period $T$, and their occurrence trigger the control tasks. The time slot allocated to a given task $T_{slot} < WCET$ is now smaller than the worst case. As usual the control signal is sent to the actuators at the end of the slot, i.e. $U(x(s_k))$ is sent at time $s_k + T_{slot}$, and the delay is equal to $T_{slot}$

$$\forall t \in [s_k + T_{slot}, s_{k+1} + T_{slot}[, \quad U = U(x(s_k)).$$

But now it may happen that occasionally a control task deadline is missed : in that case it is proposed to stop the current computation, hold the current value of the control signal for the next period and start a fresh computation with the next sensor value. In that case, the control signal $U(x(s_k))$ is hold for one extra period, i.e. if the deadline miss occur at time $s_k + T_{slot}$:

$$\forall t \in [s_k + T_{slot}, s_{k+2} + T_{slot}[, \quad U = U(x(s_k)),$$

and for $N$ consecutive deadline misses and data loss:

$$\forall t \in [s_k + T_{slot}, s_{k+N} + T_{slot}[, \quad U = U(x(s_k)).$$

In other words a newly computed control signal is sent to the actuators at non-equidistants instants $t_{k'}$ only if the control computation has been successfully carried out:

$$t_{k'} = s_k + T_{slot} \quad \text{if} \quad T_{ex} \leq T_{slot},$$

where $k'$ is a positive integer representing the number of input values which have been implemented before $s_k = kT$. Then, the control input can be asynchronous since the difference between two sampling instant $t_{k'+1} - t_{k'}$ is time-varying but bounded by $T$ and $NT$. Hence $t_{k'+1} - t_{k'} = \alpha T$, where the integer $\alpha \in [1, ..., N]$ and the asynchronous sampling is determined by the couple $(T, N)$.

As already observed and reported in aforementioned references, it is likely that a robust feedback control systems can keep stability despite occasional data loss, at the price of a decreased performance and robustness. Therefore, for a given LTI plant, a given control law, a known distribution of execution times and the weakened real-time constraint, problems to be solved can be informally stated as :

- find $N$, the maximum value of consecutive data loss before loosing the closed-loop stability;
- find an adequate value of $T_{slot}$ to fulfill a given trade-off between the control and the computing performances.
- evaluate the weakly-hard closed-loop robustness w.r.t. the plant's parameters uncertainties.

## III. Problem Formulation

Consider the linearized system representing the pitch control of a plane with a sampled and delayed input:

$$\dot{x}(t) = (A + \Delta_\mu A(t))x(t) + (B + \Delta_\mu B(t))u(t), \tag{1}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ represent the state variable and the input vector. The matrices $A$ and $B$ are constant and of appropriate dimension. The matrices $\Delta_\mu A$ and $\Delta_\mu B$ represent the uncertainties of the model which can be constant or time varying. The (time-varying) uncertainties are given in a polytopic representation:

$$\Delta_\mu A(t) = \mu \sum_{i=1}^{M} \lambda_i(t) A_i, \;\; \Delta_\mu B(t) = \mu \sum_{i=1}^{M} \lambda_i(t) B_i,$$

where $M$ corresponds to the numbers of vertices. The matrices $A_i$, $B_i$ and $C_i$ are constant and of appropriate dimension. The scalar $\mu \in R$ characterizes the size of the uncertainties. Note that when $\mu = 0$, no parameter uncertainty is disturbing the system. However the greater the $\mu$, the greater the disturbances. The functions $\lambda_i(.)$ are weighted scalar functions which follow a convexity property, i.e. for all $i = 1, .., M$ and for all $t \geq 0$, $\lambda_i(t) \geq 0$ and $\sum_{i=1}^{M} \lambda_i(t) = 1$. In this paper it is assumed that the control scheduling induces a constant transmission delay $T_{slot}$ and a sampling of the transmitted signal. For a given gain $K$ in $\mathbb{R}^{n \times m}$, the control law is a piecewise-constant static state-feedback of the form $u(t) = Kx(t_{k'} - T_{slot})$, for all $t \in [t_{k'}, \; t_{k'+1}]$. These instants $t_{k'}$ represent the instants where the control input is updated. The closed loop system is thus rewritten, for all $t \in [t_{k'}, \; t_{k'+1}]$, as

$$\dot{x}(t) = \bar{A}(t)x(t) + \bar{B}(t)Kx(t_{k'} - T_{slot}), \tag{2}$$

where $\bar{A}(t) = A + \Delta_\mu A(t)$ and $\bar{B}(t) = B + \Delta_\mu B(t)$. Several authors investigated in guaranteeing the stability of such systems. In [14], a continuous-time approach to model sampled-data systems was developed. It allows assimilating sampling effects as the ones of a particular delay. In [14], [15] or [16], the authors propose an aggregated delay formulation. They develop stability criteria which take into account the delay

$\delta$. However they did not consider the different natures of the transmission and the sampling delay. More especially the additional characteristic of sampled delay which is $\dot{\delta} = 1$ has not been included and thus leads to conservative conditions.

When $\mu$ is zero, the discrete-time modeling of such systems is easily obtained by integrating the differential equation (2) over the interval $[t_{k'},\ t_{k'}+\tau]$, for any $\tau$ in $[0,\ \bar{T}]$,

$$\begin{array}{l} x(t_{k'}+\tau) = \tilde{A}(\tau)x(t_{k'}) + \tilde{B}(\tau)Kx(t_{k'}-T_{slot}), \\ \tilde{A}(\tau) = e^{A\tau}, \quad \tilde{B}(\tau) = \int_0^\tau e^{A(\tau-\theta)}d\theta B. \end{array} \tag{3}$$

Thus, define, for all integer $k'$, the function $\chi_{k'}^{T_{slot}}$ : $[0,\ NT]\times[-T_{slot},\ 0] \to \mathbb{R}^n$ such that for all $\tau$ in $[0,\ NT]$ and all $\theta$ in $[-T_{slot},\ 0]$, $\chi_{k'}(\tau,\theta) = x(t_{k'}+\tau+\theta)$. The set $\mathbb{K}_{NT}^{T_{slot}}$ represents the set of functions defined by $\chi_k^{T_{slot}}$ as the set of continuous functions from $[0,\ NT]\times[-T_{slot},\ 0]$ to $\mathbb{R}^n$.

However, the same discretization method is not valid when the system is subject to time-varying uncertainties. Thus discrete-time analysis of (3) leads to unavoidable difficulties. Thus there is a need to introduce novel stability conditions to cope with this type of discrete-time systems.

In this paper, a novel method to assess stability of systems subject to varying sampling, constant delay and time-varying uncertainties is proposed. The main idea is to consider separately the two delays types. To do so, the stability conditions are based on the discrete-time Lyapunov Theorem but expressed with the continuous-time model of the system. It leads to less conservative necessary conditions.

## IV. Stability analysis

### A. System without uncertainties

In this section, a study on the asymptotic stability of the solutions of sampled-data systems presented in (1) with $\mu = 0$ is provided. There exist several results in the literature to ensure asymptotic stability of linear systems with input delay and sampling. In the present article, the contribution is based on the asymptotic stability conditions developed in [6].

*Theorem 1:* Consider an integer $N$ and two non negative scalars $T_{slot}$ and $T$. Assume that there exist $Q>0$, $R_1>0$ and $R_2>0 \in \mathbb{S}^n$, $P>0$, $U>0$ and $S_1 \in \mathbb{S}^{2n}$ and $S_2$ and $X \in \mathbb{R}^{2n\times 2n}$, $Y \in \mathbb{R}^{5n\times 2n}$ that satisfy for $j=1,2$ :

$$\begin{array}{l} \Psi_1(A,B) = \Pi_1(T_{slot}) + T_j\Pi_2 + T_j\Pi_3 < 0, \\ \Psi_2(A,B)\left[\begin{array}{cc} \Pi_1(T_{slot}) - T_j\Pi_3 & T_jY \\ T_jY^T & -T_jU \end{array}\right] < 0, \end{array} \tag{4}$$

where $T_1 = T$, $T_2 = NT$ and

$$\begin{array}{l} \Pi_1(T_{slot}) = 2He\{N_1^TPN_0\} + M_1^TQM_1 - M_2^TQM_2 \\ \quad +M_0^T(R_1+T_{slot}R_2)M_0 - M_{12}^TR_2/T_{slot}M_{12} \\ \quad -M_5^TR_1M_5 - N_{12}^TS_1N_{12} - 2He\{YN_{12}\} \\ \quad -2He\{N_2^TS_2N_{12}\}, \\ \Pi_2 = N_0^TUN_0 + 2He\{N_0^T(S_1N_{12}+S_2^TN_2)\}, \\ \Pi_3 = N_2^TXN_2, \end{array}$$

and

$$\begin{array}{ll} M_0 = [A\ 0\ 0\ BK\ 0], & M_1 = [I\ 0\ 0\ 0\ 0], \\ M_2 = [0\ I\ 0\ 0\ 0], & M_3 = [0\ 0\ I\ 0\ 0], \\ M_4 = [0\ 0\ 0\ I\ 0], & M_5 = [0\ 0\ 0\ 0\ I] \\ N_0 = [M_0^T\ M_5^T]^T, & N_1 = [M_1^T\ M_2^T]^T, \\ N_2 = [M_3^T\ M_4^T]^T, & M_{12} = M_1 - M_2, \\ N_{12} = N_1 - N_2. & \end{array}$$

System (2) is thus asymptotically stable for any asynchronous sampling defined by $(T,N)$ and the delay $T_{slot}$.

*Proof:* Because of space limitations, the proof is omitted. However the details can be found in [6]. ■

Note that the conditions from Theorem 1 include the robust stability properties with respect to the input delay $T_{slot}$. This means that (4) requires the system to be stable at least for the transmission delay $T_{slot}$ and $T = T_i$.

### B. System with uncertainties

In this section we will consider that $\mu \neq 0$. Then we want to extend the previous theorem to the case of time-varying uncertainties. In the previous stability theorem, the conditions depends almost linearly on the matrices defining the continuous-time model. Then the following corollary presents an extension of the previous theorem to uncertain and time-varying model.

*Corollary 1:* Consider an integer $N$ and there non negative scalars $T_{slot}$, $T$ and $\mu$. Assume that there exist the same matrices $Q$, $R_1$, $R_2$, $P$, $U$, $S_1$ and $S_2$ as in Theorem 1 and $X_i \in \mathbb{R}^{2n\times 2n}$, $Y_i \in \mathbb{R}^{5n\times 2n}$ that satisfy, for $i = 1,\dots,M$ and $j = 1,2$

$$\Psi_{1i}(A_i,B_i) = \Pi_{1i}(T_{slot}) + T_j\Pi_{2i} + T_j\Pi_{3i} < 0, \tag{5}$$

$$\Psi_{2i}(A_i,B_i) = \left[\begin{array}{cc} \Pi_{1i}(T_{slot}) - T_j\Pi_{3i} & T_jY_i \\ T_jY_i^T & -T_jU \end{array}\right] < 0, \tag{6}$$

where

$$\begin{array}{l} \Pi_{1i}(T_{slot}) = 2He\{N_1^TPN_{0i}\} + M_1^TQM_1 - M_2^TQM_2 \\ \quad +M_{0i}^T(R_1+T_{slot}R_2)M_{0i} - M_{12}^TR_2/T_{slot}M_{12} \\ \quad -M_5^TR_1M_5 - N_{12}^TS_1N_{12} - 2He\{Y_iN_{12}\} \\ \quad -2He\{N_2^TS_2N_{12}\}, \\ \Pi_{2i} = N_{0i}^TUN_{0i} + 2He\{N_{0i}^T(S_1N_{12}+S_2^TN_2)\}, \\ \Pi_{3i} = N_2^TX_iN_2, \end{array}$$

and $M_{0i}$ and $N_{0i}$ are defined as $M_0$ and $N_0$ but with the polytopes $A_i = A + \mu A_i$ and $B_i = B + \mu B_i$. The system (2) is thus asymptotically stable for for the periodic sampling defined by $T$ and the delay $T_{slot}$.

*Proof:* Consider the stability conditions from Theorem 1. By noting that

$$\begin{array}{l} M_0(t) = \left[\bar{A}(t)\ 0\ 0\ \bar{B}(t)K\ 0\right] = \sum_{i=1}^M \lambda_i(t)M_{0i}, \\ N_0(t) = \left[M_0^T(t)M_5^T\right] = \sum_{i=1}^M \lambda_i(t)N_{0i}, \end{array}$$

and by introducing the matrices variables

$$Y(t) = \sum_{i=1}^M \lambda_i(t)Y_i, \quad X(t) = \sum_{i=1}^M \lambda_i(t)X_i.$$

Then, using the Schur complement, we note that the two conditions are linear with respect to the matrices $M_{0i}$ and $N_{0i}$ so that, for $j=1,2$, $\Psi_j(\bar{A}(t),\bar{B}(t)) = \sum_{i=1}^M \lambda_i(t)\Psi_{ji}(A_i,B_i)$. This allows to conclude the proof. ■

## V. Case study

The case study applies the previous robustness approach to a weakened scheduling scheme for the pitch controller of an aircraft. We consider only the so-called "short period approximation" linearized model of an aircraft around the pitch axis. The model is given by [17]:

$$\left\{ \begin{array}{rcl} E\dot{x} & = & Fx + Gu, \\ y & = & Hx. \end{array} \right. \tag{7}$$

The state vector is $x = \begin{bmatrix} \alpha & \theta & q \end{bmatrix}$ where $\alpha$ is the angle of attack, $\theta$ is the pitch angle, $q$ is the pitch rate, the input vector $u = \delta_E$ is the elevator deflection and where

$$\begin{array}{l} E = \begin{bmatrix} V_T - Z_{\dot{\alpha}} & 0 & 0 \\ 0 & 1 & 0 \\ -M_{\dot{\alpha}} & 0 & 1 \end{bmatrix}, \; H = \begin{bmatrix} 0 & 0 & 0 & \frac{180}{\pi} \\ 0 & \frac{180}{\pi} & 0 & 0 \end{bmatrix}, \\ G = \begin{bmatrix} Z_{\delta_e} \\ 0 \\ M_{\delta_e} \end{bmatrix}, \quad F = \begin{bmatrix} Z_\alpha & -g_0' sin\gamma_e & V_T + Z_q \\ 0 & 0 & 1 \\ M_\alpha & 0 & M_q \end{bmatrix}. \end{array} \tag{8}$$

where the $E$ matrix is always non-singular in normal flight conditions. The model parameters are the dimensionless derivatives of the standard aircraft model ([17]). In practice low-pass filtering is systematically added to the noisy $\alpha$ angle of attack. State-of-the-art pitch control design also takes into account the elevator's dynamics, and an integrator is added in the feed-forward channel to ensure a zero steady-state error. The full controlled plant has an augmented state vector given by $\begin{bmatrix} \alpha & q & \delta_E & \alpha_F & \beta \end{bmatrix}$ where $\alpha_F$ is the filtered measure of $\alpha$ and $\beta$ is the output of the integrator. We can then obtain standard state space equation of the form:

$$\left\{ \begin{array}{l} \dot{x} = Ax + Bu, \\ y = Cx, \end{array} \right. \tag{9}$$

where $A = E^{-1}F$ and $B = E^{-1}G$. In our case study, we have considered the F16 aircraft with the flight conditions given in table I. The nominal condition is: $h = 0ft$, $x_{cg} =$

| Variable | Nominal | $x_{cg} = 0.3\bar{c}$ | $x_{cg} = 0.38\bar{c}$ |
|---|---|---|---|
| $V_T$ (ft/s) | 502.0 | 502.0 | 502.0 |
| $\alpha$(rad) | 0.03691 | 0.03936 | 0.03544 |
| $\theta$(rad) | 0.03691 | 0.03936 | 0.03544 |
| Q (rad/s) | 0 | 0 | 0 |
| Thtl (0-1) | 0.1385 | 0.1485 | 0.1325 |
| El (deg) | -0.7588 | -1.931 | -0.05590 |

TABLE I
FLIGHT CONDITIONS FOR SIMULATIONS

$0.35\bar{c}$, $\dot{\theta} = 0$. Using the nominal condition and a flight at sea level, we obtain the following numerical values for the pitch model:

$$\begin{array}{l} A = \begin{bmatrix} -1.01887 & 0.90506 & -0.00215 & 0 & 0 \\ 0.82225 & -1.07741 & -0.17555 & 0 & 0 \\ 0 & 0 & -20.2 & 0 & 0 \\ 10.0 & 0 & 0 & -10 & 0 \\ 0 & -57.2958 & 0 & 0 & 0 \end{bmatrix}, \\ B = \begin{bmatrix} 0 \\ 0 \\ 20.2 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 0 & 57.2958 & 0 \\ 0 & 57.2958 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \end{array} \tag{10}$$

An output feedback controller $u = Ky = -k_\alpha \alpha_F - k_q q - k_i \beta$ is designed using standard pole placement giving $K = [-0.04238; -0.4098; 0.8426]$. The control period is chosen as $T = 0.08$ sec, and the nominal computing slot is chosen as $WCET = 0.02$ sec (considering that the CPU is shared by four control tasks). The matrices $A_i$ and $B_i$, elements of the convex combination $\Delta_\mu A(t)$ and $\Delta_\mu B(t)$ are defined in a simple 2 vertices polytope as $A_i = (-1)^i A$ and $B_i = (-1)^i B$ for $i = 1, 2$. Starting from the initial 'hard real-time' pattern described in Figure 1, where a $WCET$ time slot is allocated to the control task with a period $T$, two weakened scheduling schemes have been considered (Figure 4):

- **Case 1:** A time slot $T_{slot} < WCET$ is allocated to the control task, but the system's period $T$ keeps its initial value. In that case, some extra time remains to compute other activities before the new control activation. Some control improvement can be expected from the reduction of the latency from $WCET$ to $T_{slot}$;
- **Case 2:** Again, a time slot $T_{slot} < WCET$ is allocated to the control task, but the system's period is now also reduced by the same value, while the time remaining for computing other activities remains $T_{others}$ as in the initial scheme. In that case control improvement is expected from both the latency and sampling period reduction.

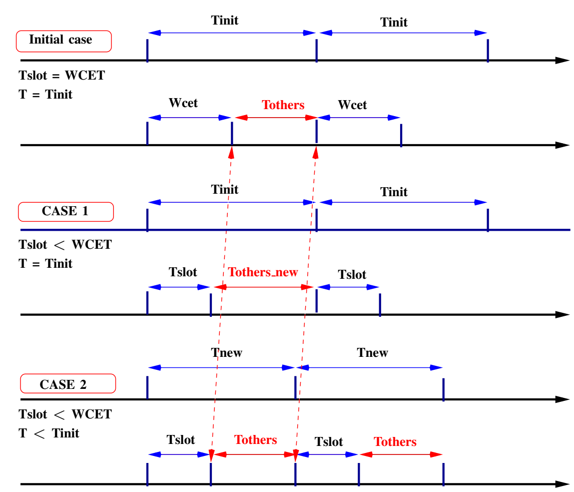


Fig. 4. Scheduling cases

The stability conditions of Theorem 1 are used to find the relations between the computing slot value (given by the ratio $\epsilon = \frac{T_{slot}}{WCET}$ and the maximum number of consecutive deadlines misses before unstability $N$, for the two cases and for several values of $\mu$. The analysis of Case 1 (top of Figure 5) shows that the tolerance of the controller w.r.t. deadlines misses, measured by $N$, increases when $T_{slot}$ is decreased (therefore also decreasing the systematic latency). It also shows that increasingly uncertain systems, with growing values of $\mu$, are less tolerant w.r.t. deadlines misses. In Case 2, (bottom of Figure 5) the number $N$ of sustainable consecutive deadline misses is even improved, as the reduction of $T_{slot}$ induces a decreasing in both the delay and the sampling interval.

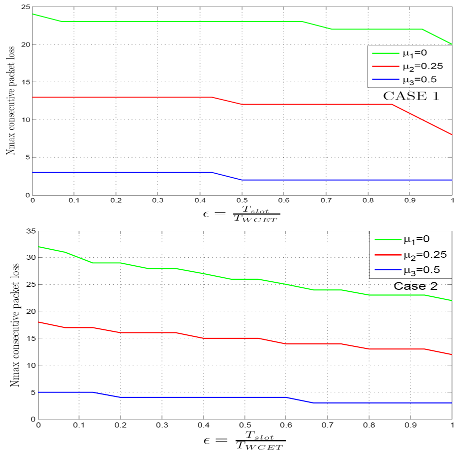


Fig. 5. Maximum allowed consecutive deadline misses

Nevertheless decreasing $T_{slot}$ obviously increases the risk of missing deadlines. For a given distribution of execution times, the probability of missing deadlines decreases from 1 to 0 as the scheduling factor $\epsilon$ decreases from 1 ($T_{slot} = WCET$) to a minimum value where $T_{slot} = BCET$ (Best Case Execution Time), as depicted by the black plot in Figure 6. Assuming that the execution times of the task instances are independent, the probability of reaching the maximum tolerable number of consecutive misses are given in the same figure for the two cases and for different values of $\mu$. Hence, for a given scheduling scheme and uncertainty assumption, it is easily possible to compute the scheduling factor $\epsilon$ corresponding at a given failure probability.

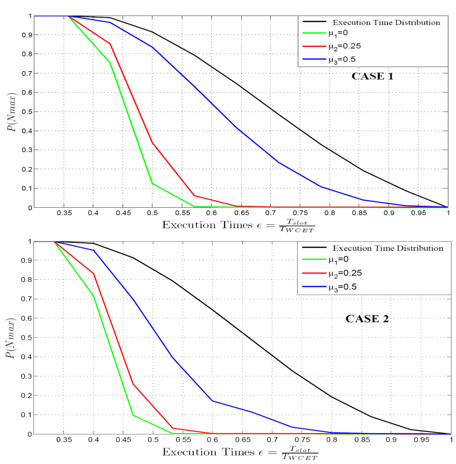


Fig. 6. Probability of reaching the maximum tolerable consecutive deadline misses

## VI. Conclusion

In this paper the hard real-time assumption has been revisited based on robustness considerations. The theoretic contribution provides new stability conditions for feedback linear systems submitted to delays, varying sampling and uncertainties. In the case of a control task miss its deadline, the computation should be aborted and the preceding control signal hold for an extra control period, leading to a varying sampled system. In this framework, the stability condition allows computing the maximum number of consecutive deadline misses which can be tolerated by an uncertain system while keeping stability. Future design and implementation rules may be investigated to find cost effective trade-offs between embedded computing power, control performance, control robustness and overall fault-tolerance.

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