LPV/$\mathcal{H}_\infty$ control of an Autonomous Underwater Vehicle (AUV)

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Abstract— **This paper deals with the robust control of an Autonomous Underwater Vehicle (AUV), using the $\mathcal{H}_\infty$ approach for Linear Parameter Varying (LPV) polytopic systems. This method seems to be adapted to this kind of system because of the important non-linearities and the uncertainties on the hydrodynamics parameters.**

After the presentation of the nonlinear model of the AUV considered for the study, an LPV model was built, regarding the mass of the vehicle as unique varying parameter. Then the methodology of the control law applied is exposed and simulation results are presented. In particular, a comparison with the $\mathcal{H}_\infty$ approach will show the interest of the method in terms of control performance.

I. INTRODUCTION

For system with important non-linearities and large uncertainties on parameters, the use of a robust control, such as the $\mathcal{H}_\infty$, may not be sufficient to obtain good performances for the closed loop system over all the parameters variation range. The Linear Parameter Varying (LPV) theory, based on robust control, improves the method by taking into account the non-linearities and variations of the system at design time [1]. The obtained controller is then scheduled by a set of parameters, and is on-line adapted at the working points.

In this paper the $\mathcal{H}_\infty$ approach for LPV system will be applied to an Autonomous Underwater Vehicle (AUV). The use of AUV for the exploration of seabed, and the control of these vehicles has been of large interest for researcher in the past two decades. Many different control laws were studied along the years : decoupling steering, diving, and speed control by PID [2], coupled PID and anti-windup control [3], sliding mode control [4], [5] and $\mathcal{H}_\infty$ control [6], [7].

The control of these vehicles is made difficult by numerous non-linearities, due to cross-coupled dynamics and hydrodynamic forces. Moreover the knowledge about the vehicle parameters is very poor : it may reach up to 70% ([8]) for the off-line estimation of hydrodynamics parameters. On the other hand the performance required by the payload may be high, e.g. the roll and pitch velocities must be kept inside tight bounds to perform high quality imaging using a side-scan sonar.

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Hence robust control is necessary to perform safely autonomous missions : in this paper we use the ability of LPV based control to combine performance specification in the $\mathcal{H}_\infty$ framework and the extended adaptation to parameters variations inside a specified range provided by the LPV approach.

Fig. 1. The $Aster^X$ AUV operated by Ifremer

The AUV considered in this paper is an $Aster^X$ like vehicle, developed by the Ifremer (French Research Institute for Exploitation of the Sea)[1]. It is actuated by a main screw propeller for moving in the longitudinal direction. Two fins in the front part of the vehicle (canard fins), and two couples of fins at the tail of the vehicle (horizontal and vertical plan) allow for the steering of the vehicle around its roll, pitch and yaw angles. The AUV has several navigation sensors : a Doppler loch to measure the speed, an inertial system (using gyroscope, accelerometers and magnetometers) to compute in real time its attitude (roll, pitch and yaw angle) and update its position, and also an acoustic sensor for (low rate) absolute positioning.

The mission considered here is the cartography of sea floor. In terms of control objectives, that means moving at constant speed, at a constant altitude with respect to the sea floor, with constrained roll and pitch velocities, so that the reconstruction of the map by collected data will be possible with limited post-processing.

For this mission, only motions in the vertical plan will be considered : the control of the yaw angle and speed will not be taken into account.

The paper is organized as follows. Section 2 presents the nonlinear model of the AUV considered for the study, and also the derived LPV model. Section 3 explains the

[1] `http://www.ifremer.fr/fleet/r&dprojets.htm`

computation of the LPV controller that will be applied on the system (definition of the structure and weighting function, as well as the computational point of view). Finally, section 4 contains simulation results of the LPV controller, and also a comparison with an $\mathcal{H}_\infty$ controller designed with the same specifications.

II. AUV MODEL

The model description of the autonomous underwater vehicle (AUV), is based on two referentials :

- The referential linked to the vehicle : $\mathcal{R}(C, X, Y, Z)$ (the origin C is the hull center).
- The inertial referential : $\mathcal{R}_0(O, X_0, Y_0, Z_0)$. This referential can be taken as linked to the earth in the case of AUV moving at slow speed.

For the description of the vehicle behavior, we consider a 12 dimensional state vector : $X = \begin{bmatrix}\eta(6) & \nu(6)\end{bmatrix}^T$.

$\eta(6)$ is the position, in the inertial referential $\mathcal{R}_0$, describing the linear position η_1 and the angular position η_2: $\eta = \begin{bmatrix}\eta_1 & \eta_2\end{bmatrix}^T$ with $\eta_1 = \begin{bmatrix}x & y & z\end{bmatrix}^T$ and $\eta(2) = \begin{bmatrix}\phi & \theta & \psi\end{bmatrix}^T$ where x, y and z are the positions of the vehicle in the inertial referential : $\mathcal{R}_0(O, X_0, Y_0, Z_0)$, and ϕ, θ and ψ are respectively the roll, pitch and yaw angle.

$\nu(6)$ represent the velocity vector, in the local referential $\mathcal{R}$ describing the linear and angular velocities (first derivative of the position, considering the change of referential, see equation (2)) : $\nu = \begin{bmatrix}\nu_1 & \nu_2\end{bmatrix}^T$ with $\nu_1 = \begin{bmatrix}u & v & w\end{bmatrix}^T$ and $\nu_2 = \begin{bmatrix}p & q & r\end{bmatrix}^T$

As given in [9], [8], [10], the physical model is given by the following dynamical equation:

$$M\dot{\nu} = G(\nu)\nu + D(\nu)\nu + \Gamma_g + \Gamma_p + \Gamma_u \qquad (1)$$

$$\dot{\eta} = J_c(\eta_2)\nu \qquad (2)$$

where:

- M is the inertial matrix. It contains the real mass of the vehicle augmented by the "water-added-mass" part,

- $G(\nu)$ represents the action of Coriolis and centrifugal forces,

- $D(\nu)$ is the matrix of hydrodynamics damping coefficients,

- Γ_g correspond to the gravity effort and hydrostatic force,

- Γ_p represents disturbing forces and moments (e.g. due to waves, ocean currents…)

- $J_c(\eta_2)$ is the matrix for the change of referential. $\mathcal{R}(C, xyz)$ towards $\mathcal{R}_0(O, X_0Y_0Z_0)$,

- Γ_u represent the forces and moments due to vehicle actuators. The considered AUV has a propeller for the control of velocity in Ox direction (forward force Q_c) and 3 pairs of fins :

- 2 horizontals fins in the front part of the vehicle (controlled with an angle β_1)
- 2 horizontals fins at the tail of the vehicle (controlled with an angle β_2)
- 2 verticals fins at the tail of the vehicle (controlled with an angle δ_1)

The model obtained is nonlinear and includes 12 state variables and 4 controls inputs. For the computation of the controller, a linear model is proposed. The equilibrium point is chosen as $[u\ v\ w\ p\ q\ r] = [1\ 0\ 0\ 0\ 0\ 0]$: all velocities are taken equal to 0, except the longitudinal velocity taken equal to 1m/s, the cruising speed chosen by the operator according to the payload requirements.

Tangential linearisation around the chosen equilibrium point yields to a model of the form :

$$\begin{cases} \dot{x} = Ax(t) + Bu(t) \\ y = Cx(t) + Du(t) \end{cases}$$

where

- x stand for the state : $x = [x\ u\ y\ v\ z\ w\ \phi\ p\ \theta\ q\ \psi\ r]^T$
- u for the control input $u = [\beta_1\ \beta_2\ \delta_1\ Q_c]^T$
- y for the measured output

All the matrices A, B, C and D depend on the model parameters : hydrodynamical parameters, mass of the vehicle, dimension of fins…Note that most of these parameters are uncertain.

In this preliminary approach, the mass of the vehicle M is chosen below as the unique varying parameter. Indeed, the mass is one of the parameter varying during the navigation (because of the water-added terms uncertainty, or due to the casting off of payloads during a mission). The choice of this unique varying parameters allows for keeping the controller reconstruction simple. More complex sets of varying parameters could be chosen in future studies to enlarge the set of operating conditions, such as the speed of the vehicle like in [11] for missile control. However, the considered application (cartography) allows to assume almost constant speed.

The behavior of the non linear model of the AUV is then approximated by a Linear Parameter Varying (LPV) system, where the matrices of A, B, C and D are linearly dependent of a set of parameters θ as shown by the equation :

$$\begin{cases} \dot{x} = A(\theta(\cdot))x(t) + B(\theta(\cdot))u(t) \\ y = C(\theta(\cdot))x(t) + D(\theta(\cdot))u(t) \end{cases}$$

When parameters can be bounded by $\underline{\theta}$ (minimum value of the parameter θ) and $\overline{\theta}$ (maximum value), a controller can be computed, valid for all variation of $\theta \in [\underline{\theta}, \overline{\theta}]$. Here a polytopic model has been used : a polytope (convex polyhedron) is constructed by tacking combination of bounds of parameters as vertices. In the case of a single varying parameter θ, we have 2 vertices : $\underline{\theta}$ and $\overline{\theta}$.

As the model obtained by linearisation is already linear according to the mass, only the equations of the model at the vertices of the polytope are required (corresponding to the bounds of the variation of the parameter).

The system G, corresponding to a mass M belonging to the interval $[\underline{M}, \overline{M}]$ can be given by the convex combination of G_{min} and G_{max} :

$$\begin{cases} G_M = \alpha_1 \times G_{min} + \alpha_2 \times G_{max} \\ \text{with} \\ \alpha_1 = \frac{\overline{M}-M}{\overline{M}-\underline{M}} \; and \; \alpha_2 = \frac{M-\underline{M}}{\overline{M}-\underline{M}} \end{cases}$$

The model obtained is polytopic : a controller adapted to this kind of system has to be computed for the mission considered (to follow the see bottom at constant altitude).

III. LPV CONTROLLER

In this section a LPV controller will be designed for the above LPV model.

A. Structure and weighting function

The method is based on the $\mathcal{H}_\infty$ control design. The first step is to choose a structure and weighting functions that will be placed in the control loop for setting some specifications (response time in closed loop, tracking error...).

We choose the following classical structure, with :

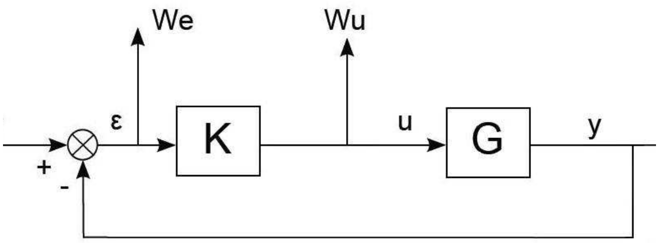

Fig. 2. Structure chosen for the control design

- W_e a weight on the tracking error, for fixing specifications on the controlled outputs (u and z) :

$$\frac{1}{W_{e_{u,z}}} = \frac{s + w_b \epsilon}{\frac{s}{M_s} + w_b}$$

  with
  - $M_s = 2$ for a good robustness margin.
  - $\epsilon = 0.01$ so that the tracking error will be less that 1%.
  - $w_b = 0.46$ for having a response time of 5 seconds.
- W_u is chosen to account for actuator limitations (all action where normalized, so we choose the identity matrix of size 4 for W_u).
- W_y to restrict the evolution of q (pitch speed) and p (roll speed), to help the post-processing reconstruction of sea bottom map. W_y is chosen as $W_{y_{p,q}} = 10^{-2}$.

Then the problem is rewritten in the standard form (Fig 3) : This LFT formulation allows to study the transfer function between w (exogenous inputs : reference and disturbance) and z (controlled output), y are the measured output and u the control input. P is the augmented plant : it contains the model of the system and the weighting functions.

To solve the problem, the $\mathcal{H}_\infty$ control approach for LPV polytopic systems is considered, as described in the next section.

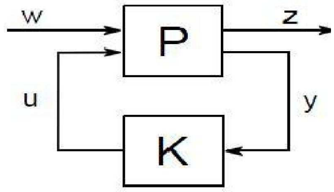

Fig. 3. Problem on standard form

B. $\mathcal{H}_\infty$ control for LPV polytopic systems

A dynamical LPV system can be described in the following form:

$$\Sigma(\theta) : \begin{bmatrix} \dot{x} \\ \hline z \\ y \end{bmatrix} = \left[\begin{array}{c|cc} A(\theta) & B_1(\theta) & B_2(\theta) \\ \hline C_1(\theta) & D_{11}(\theta) & D_{12}(\theta) \\ C_2(\theta) & D_{21}(\theta) & D_{22}(\theta) \end{array}\right] \begin{bmatrix} x \\ \hline w \\ u \end{bmatrix} \tag{3}$$

where x define the state,, w, u, z and y as previously seen. $\theta(.) \in \Theta$ is the set of varying parameters that describe a set of systems. $A \in R^{n\times n}$, $B_1 \in R^{n\times n_w}$, $B_2 \in R^{n\times n_u}$, $C_1 \in R^{n_z\times n}$, $D_{11} \in R^{n_z\times n_w}$ and $D_{12} \in R^{n_z\times n_u}$, $C_2 \in R^{n_y\times n}$, $D_{21} \in R^{n_y\times n_w}$ and $D_{22} \in R^{n_y\times n_u}$.

A LPV controller is defined by:

$$S(\theta) : \begin{bmatrix} \dot{x}_c \\ u \end{bmatrix} = \begin{bmatrix} A_c(\theta) & B_c(\theta) \\ C_c(\theta) & D_c(\theta) \end{bmatrix} \begin{bmatrix} x_c \\ y \end{bmatrix} \tag{4}$$

where x_c, y and u are the state, the exogenous input and controlled output respectively of the controller associated to the system (3). $\theta(.) \in \Theta$ is the set of the varying parameters associated to the controller. $A_c \in R^{n\times n}$, $B_c \in R^{n\times n_y}$, $C_c \in R^{n_u\times n}$ and $D_c \in R^{n_u\times n_y}$.

The LPV closed-loop system is defined by:

$$CL(\theta) : \begin{bmatrix} \dot{\eta} \\ z \end{bmatrix} = \begin{bmatrix} \mathcal{A}(\theta) & \mathcal{B}(\theta) \\ \mathcal{C}(\theta) & \mathcal{D}(\theta) \end{bmatrix} \begin{bmatrix} \eta \\ w \end{bmatrix} \tag{5}$$

where η, w and z are the state, the input and output respectively of the closed-loop system. $\mathcal{A} \in R^{n\times n}$, $\mathcal{B} \in R^{n\times n_w}$, $\mathcal{C} \in R^{n_z\times n}$ and $\mathcal{D} \in R^{n_w\times n_z}$ depend on θ.

The aim of the LTI/$\mathcal{H}_\infty$ synthesis is to minimize the $\mathcal{H}_\infty$ norm (γ_∞) of a system ensuring the internal stability ($K > 0$). From the linear dissipative systems theory, finding such a controller leads to the Bounded Real Lemma (BRL) given by inequality (6) [12], [13]see.

$$\begin{bmatrix} \mathcal{A}^T K + K\mathcal{A} & K\mathcal{B} & \mathcal{C}^T \\ \mathcal{B}^T K & -\gamma_\infty^2 I & \mathcal{D}^T \\ \mathcal{C} & \mathcal{D} & -I \end{bmatrix} < 0 \tag{6}$$

(6) is a Bilinear Matrix Inequality (BMI) so far, hence a non-convex problem has to be solved. Via a change of basis expressed in [13], and assuming $D_{22} = 0$ (in (3)), we can find a non-conservative LMI (7) that expresses the same problem in a tractable way for Semi-Definite Programs (SDP),

$$
\begin{bmatrix}
A\mathbf{X}+\mathbf{X}A^T+B_2\widetilde{\mathbf{C}}+\widetilde{\mathbf{C}}^T B_2^T & * & * & * \\
\widetilde{\mathbf{A}}+A^T+C_2^T\widetilde{\mathbf{D}}^T B_2^T & \mathbf{Y}A+A^T\mathbf{Y}+\widetilde{\mathbf{B}}C_2+C_2^T\widetilde{\mathbf{B}}^T & * & * \\
B_1^T+D_{21}^T\widetilde{\mathbf{D}}^T B_2^T & B_1^T\mathbf{Y}+D_{21}^T\widetilde{\mathbf{B}}^T & -\gamma I_m & * \\
C_1\mathbf{X}+D_{12}\widetilde{\mathbf{C}} & C_1+D_{12}\widetilde{\mathbf{D}}C_2 & D_{11}+D_{12}\widetilde{\mathbf{D}}D_{21} & -\gamma I_q
\end{bmatrix} < 0
\qquad (7)
$$

$$
\begin{bmatrix} \mathbf{X} & I_n \\ I_n & \mathbf{Y} \end{bmatrix} > 0
$$

Solving (7) leads to the $\mathcal{H}_\infty$ optimal solution. Then the controller is obtained solving:

$$
\begin{cases}
\widetilde{D} &= D_c \\
\widetilde{C} &= D_c C_2 X + C_c M^T \\
\widetilde{B} &= Y B_2 D_c + N B_c \\
\widetilde{A} &= YAX + YB_2 D_c C_2 X + NB_c C_2 X \\
 &+ \; YB_2 C_c M^T + NA_c M^T
\end{cases}
$$

In the LPV framework, the controller has to reach these objectives for the whole set of varying parameters. Hence the previous BRL (6) becomes,

$$
\begin{bmatrix}
\mathcal{A}(\theta)^T K + K\mathcal{A}(\theta) & K\mathcal{B}(\theta) & \mathcal{C}(\theta)^T \\
\mathcal{B}(\theta)^T K & -\gamma_\infty^2 I & \mathcal{D}(\theta)^T \\
\mathcal{C}(\theta) & \mathcal{D}(\theta) & -I
\end{bmatrix} < 0 \qquad (8)
$$

that can be turned into an LMI in the same way as described in equation (7), where θ holds for the varying parameters. As θ is varying between upper ($\overline{\theta}$) and lower ($\underline{\theta}$) bounds, the LMI based problem results in an infinite set of LMIs to solve. Different approaches to reduce this problem into a finite number of LMIs are commonly used:

1) Griding parameter space
2) Linear Fractional Transformation (LFT)
3) Polytopic set of parameters

In this paper the polytopic approach is used, which is appropriate when the parameter dependency enters in a linear way in the system definition, and when the number of varying parameters is small. Applied to the $\mathcal{H}_\infty$ problem, such an approach consists in finding a common parameter independent Lyapunov function $K > 0$ and a minimal γ_∞ that solve the previous LMI problem at each vertex of the polytope defined by system (3). Then the applied control is a convex combination of these controllers and can be expressed as follows:

$$
S(\theta) = \sum_{k=1}^{2^i} \alpha_k(\theta) \begin{bmatrix} A_{c_k} & B_{c_k} \\ C_{c_k} & D_{c_k} \end{bmatrix} \qquad (9)
$$

where,

$$
\alpha_k(\theta) = \frac{\prod_{j=1}^{i} |\theta(j) - \mathcal{C}^c(\Theta_k)_j|}{\prod_{j=1}^{i} (\overline{\theta}(j) - \underline{\theta}(j))} \qquad (10)
$$

and,

$$
\sum_{k=1}^{2^i} \alpha_k(\theta) = 1 \; , \; \alpha_k(\theta) > 0 \qquad (11)
$$

where i is the number of varying parameters and $k = 2^i$, the number of vertices of the polytope. Finally, $\mathcal{C}^c(\Theta_k)$ represents the complementary of Θ_k, which is simply the k^{th} vertex of the polytope.

Note that the size of the resulting controller is the same as the original $\mathcal{H}_\infty$ controllers at the vertices, and that the on-line overhead is due to the computation of the $\alpha_k(\theta)$ polytopic coordinates of the varying parameter and to the weighted sum of the vertex controllers, and that only one off-line synthesis is needed to handle all the variation range of the considered parameter. Moreover the LPV approach ensures the stability of the closed-loop systems whatever are the variation speeds of the parameters inside the specified polytope, which is not secured when using a bank of controllers synthesised for discrete values of the parameters.

By solving the $\mathcal{H}_\infty$ problem for the LPV system using the *Yalmip* interface and the *Sedumi* solver we obtain $\gamma_{opt} = 10454$. This large value of γ is due to the behavior of the system at very low pulsations ($\leq 10^{-8}$), so the influence on the time response will be limited.

IV. Simulation results

A. $\mathcal{H}_\infty$ controller

First, a $\mathcal{H}_\infty$ controller is synthesized at the nominal mass. The considered mission consists in moving at constant speed while following the sea bottom at a constant height. The simulations are made on both the linearized and nonlinear systems : the linear system is the one used for the synthesis and serves as reference, while the nonlinear one is more realistic and represents the real system, this second model will be used to validate the control approach. For the comparison, the same structure and weighting functions are considered (see Subsection III-A). Figures 4 and 5 show the results of simulation for the control of the cruising speed u and of the altitude z for the linear and nonlinear systems, using the $\mathcal{H}_\infty$ controller.

The results obtained are quite good : however the longitudinal speed u presents oscillations during altitude variations (due to the rejection of the perturbation on u due to the trail effort when z varies) and there is an overshoot on z at changes of the requated slope. Above all the response time is too slow to reach the specified performance.

B. LPV controller synthetized for 20% of mass variation

The LPV controller obtained in section III-B is applied on the AUV which mass is fixed at the nominal value.

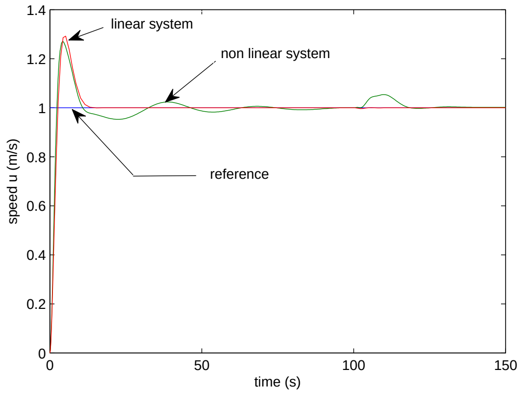

Fig. 4. Longitudinal speed u, $\mathcal{H}_\infty$ controller

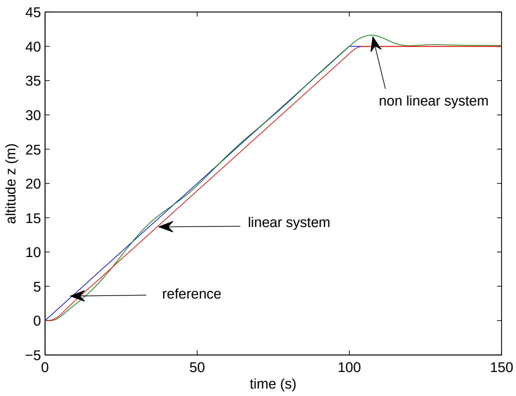

Fig. 5. Altitude z, $\mathcal{H}_\infty$ controller

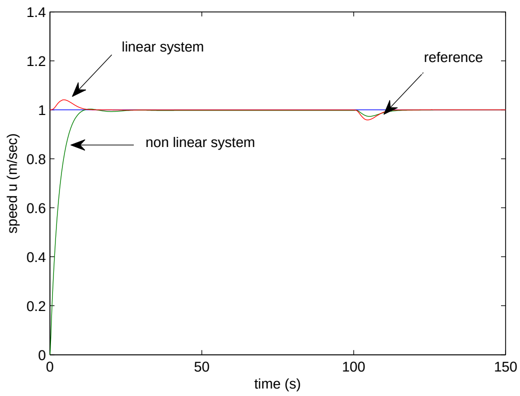

Fig. 6. Longitudinal speed u, LPV controller, nominal mass

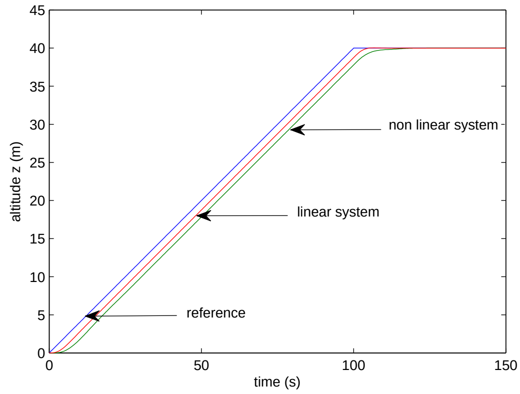

Fig. 7. Altitude z, LPV controller, nominal mass

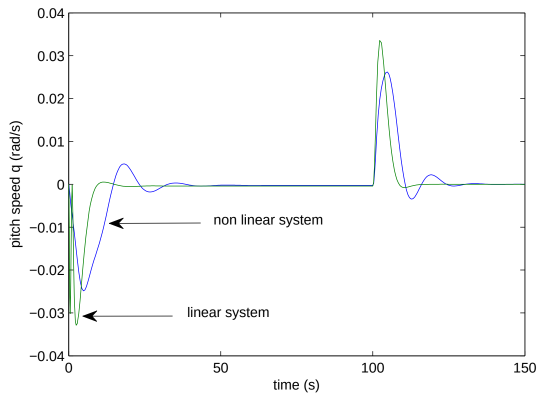

Fig. 8. Pitch speed q, LPV controller, nominal mass

Figures 6 and 7 show the simulation results for the control of the cruising speed u and of the altitude z for the linear and nonlinear systems with the nominal mass, using the LPV controller synthesized for mass variations between $+$ and $-$ 20% of the nominal value.

Conversely with the previous (purely robust) case the specifications fixed for the synthesis are now reached. For the longitudinal speed, the response time fits to the one fixed at design step. On the nonlinear model, the change of reference on z (which influences u because of the trail effort) are well absorbed and the static error is less than 0.1%. The altitude z follows the reference fixed (the slope considered here is the largest one that can be achieved considering the fins actuators without stalling the AUV), while the pitch speed is small enough for the map reconstitution (see figure 8. Moreover, the actuator's limits are respected.

C. *Adaptation of the controller at the working point*

The LPV approach requires an on-line measure or estimation of the varying parameter (here the mass). We have seen that, thanks to the robustness brought by the method, the LPV controller synthetized for the nominal mass works without adaptation with little loss of performances provided that the mass uncertainty around the nominal value is small enough ((+ or $-$ 20%).

To improve these performances, the adaptation of the controller w.r.t. to the working point is implemented. We consider the case of a sudden change of mass at $50s$ (e.g. casting off a known payload) from $800kg$ to $640kg$. Figures

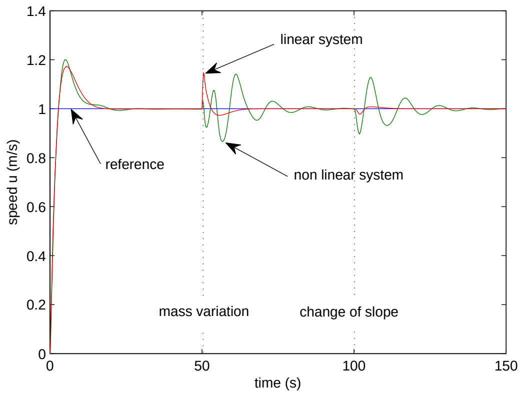

Fig. 9. Longitudinal speed u, LPV controller, adaptation to mass variation

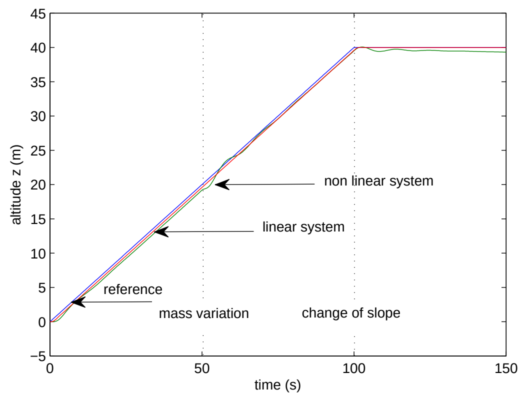

Fig. 10. Altitude z, LPV controller, adaptation to mass variation

9 and 10 show the simulation results. The performance are still correct, whereas the use of the pevious $\mathcal{H}_\infty$ controller leads to the instability of system.

V. CONCLUSIONS AND PERSPECTIVES

In this paper, an LPV controller was designed and then applied to an Autonomous Underwater Vehicle (AUV) operated in bottom following mode. The main interest of the method is its robustness w.r t. the variations of the mass of the vehicle, chosen here as the varying parameter. Indeed by considering small variations around the nominal mass, the nominal controller (synthesized at the nominal mass) robustly works without adaptation at the price of a deterioration of performances. The adaptation to the value of the varying parameter lead to even better results.

Along these preliminary experiments, several combinations of exogenous inputs and controlled outputs have been tested (although not all reported here). Unexpected difficulties often occurred during control synthesis (as for example illustrated by the large value of γ_{opt} reported in section III-B). In other cases the synthesized controller behaves well to control the linearized system, and fails to control the non-linear plant. In fact such an AUV controlled by fins has weak controllability properties for some directions, e.g. motions along the z and around the pitch axis are not separable. It is believed that some of the problems encountered came from that some information about the complex coupling between the degrees of freedom and the actuators of the AUV is lost during the linearisation process. Conflicting constraints may also arise from the set of clumsy chosen control objectives, in particular regarding the under-actuated nature of the vehicle.

However these first encouraging results foster ongoing research to better understand how the LPV approach can be used to efficiently and robustly control such autonomous vehicle. In particular the control objectives deserve to be more accurately captured taking account the controllability properties of the vehicle. These enhancements will be necessary to fully control the AUV, involving even more complex dynamics and cross coupling, e.g. to handle more complex missions like 3D following a concentration gradient to localize an emissive source.

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