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On the Fragility of PI Controllers for Time-Delay SISO Systems

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Abstract— **This paper focuses on the fragility analysis of PI-controllers for single-input-single-output (SISO) systems subject to input (or output) delays. Using a geometric approach, we present a simple and user-friendly approach not only to analyze the fragility of PI controllers, but also to provide practical guidelines for the design of *non-fragile* PI controllers. The proposed methodology is illustrated by analyzing several examples encountered in the control literature.**

Index Terms— **PI-controller, Stability, Fragility, Delay.**

I. INTRODUCTION

As reported in the literature [25], [27], more than 98% of the control-loops in the paper industries are controlled by SISO PI controllers. The "popularity" of PI and PID controllers can be attributed to their particular distinct features: simplicity and easy implementation. A long list of PI and PID tuning methods for controlling processes can be found in [25], [2]. As mentioned by [1], such controllers have to be designed by considering: (a) *performance* criteria; (b) *robustness* issues and, finally, (c) *fragility*. Roughly speaking, a controller for which the closed-loop system is destabilized by small perturbations in the controller parameters is called *"fragile"*. In other words, the fragility describes the deterioration of closed-loop stability due to small variations of the controller parameters.

This paper focuses on the fragility of PI controllers for SISO systems in the presence of I/O delays. The problem received a lot of attention in delay free systems, see, e.g., [16] (robustness techniques design leading to fragile controllers), [10] (non-fragile PID control design procedure), [1] (appropriate index to measure the fragility of PID controllers). In this context of delay free systems, some remarks concerning the controller robustness via coprime factorization and robustness optimization tools can be found in [17], [15]. However, there exists only a few results in the delay case: [28], where only (stable) first-order systems were considered, and more recently, [18], where the authors proposed a robust non-fragile control design for a TCP/AQM models and, to the best of the authors' knowledge, there does not exist any complete characterization of the fragility of PI controllers.

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In this paper, we develop a simple method to analyze the fragility of a given PI-controller for *any* SISO system subject to (constant) time-delay. The method is based on two "ingredients": (i) the construction of the *stability crossing curves* in the parameter-space defined by "P" (proportional) and "I" (integral) coefficients, and (ii) the explicit computation of the distance of some point to the closest stability crossing curves by taking into account the smoothness properties of the curves. The first step sends back to the D-decomposition method suggested by Neimark [24] in the 40s (see [19] for further comments). More precisely, the stability crossing curves represent the collection of all points for which the corresponding characteristic equation of the closed-loop system has roots on the imaginary axis. These curves define a "partition" of the space of parameters in several regions, each region having a constant number of unstable roots for all the parameters inside the region. Next, by taking into account the crossing boundaries characterization in the controller parameter-space we derive an *algorithm* allowing us to determine *explicitly* the *optimal non-fragile controller*. In other words, we present an algorithm that allows to explicitly compute the (closed-loop) *stability radius* in the controller parameter space.

Finally, as a by-product of the analysis, we can easily derive the *maximum controller gain interval* guaranteeing the closed-loop stability for a prescribed integral coefficient. Such a geometrical argument completes in the SISO framework the results [31], [26] based on the small-gain theorem.

The remaining part of the paper is organized as follows: some preliminary results are briefly presented in Section 2. Next, the fragility algorithm for PI controllers is described in Section 3 and some illustrative examples are considered in Section 4. Concluding remarks end the paper.

II. PRELIMINARY RESULTS

Consider now the class of *strictly proper* SISO open-loop system with I/O delays given by the transfer function:

$$H_{yu}(s) = \frac{P(s)}{Q(s)} e^{-s\tau} = c^T (sI_n - A)^{-1} b e^{-s\tau} \qquad (1)$$

where (A, b, c^T) is a state-space representation of the open-loop system. The control law is defined by a classical PI controller $K(s)$ of the form:

$$K(s) = k \left(1 + \frac{1}{T_i s} \right) = k_p + \frac{k_i}{s}. \qquad (2)$$

Therefore, the stability of the closed-loop systems is given by the locations of the zeros of the following meromorphic function $H : \mathbb{C} \times \mathbb{R}^2 \times \mathbb{R}_+ \mapsto \mathbb{C}$ given by:

$$H(s; k_p, k_i, \tau) = 1 + \frac{P(s)}{Q(s)}\left(k_p + \frac{k_i}{s}\right)e^{-s\tau} \qquad (3)$$

which has an infinite (countable) number of roots (see, e.g., [6], [9]).

As mentioned in the Introduction, the goal of this paper is to derive an *appropriate PI controller* (k_p^*, k_i^*) and a positive value d such that the control law (2) stabilizes the system (1) for any k_p and k_i as long as

$$\sqrt{(k_p - k_p^*)^2 + (k_i - k_i^*)^2} < d.$$

For the brevity of the paper and without any loss of generality, we make the following:

Assumption 1: The polynomials $P(s)$, $Q(s)$ in (3) are such that $P(s)$ and $sQ(s)$ do not have common zeros.

If the condition above is violated, two situations may occur: (a) $P(0) = 0$, or (b) there exists a nontrivial factor $c(s)$ ($\neq$ constant) such that $P(s) = c(s)P_1(s)$ and $sQ(s) = sc(s)Q_1(s)$. Consider first $P(0) = 0$. It is easy to see that P rewrites as $P(s) = sP_0(s)$ and (3) becomes:

$$H(s; k_p, k_i, \tau) = 1 + \frac{P_0(s)}{Q(s)}(k_p s + k_i)e^{-s\tau},$$

which corresponds to a delayed SISO system subject to a PD controller. As expected, our method works also in such a situation but this analysis is omitted. In the second case, by simplifying by $c(s)$, we obtain a system described by (3) which satisfies the Assumption above.

In the sequel, we recall some geometric results that enable us to generate the *stability crossing curves* in the space defined by the controller's parameters (k_p, k_i) (similar results for different types of dynamics can be found in [7], [20], [23]). These curves represent the collection of all pairs (k_p, k_i) for which the characteristic equation (3) has at least one root on the imaginary axis of the complex plain.

According to the continuity of zeros with respect to the system parameter (see, for instance, [4] for the continuity with respect to delays), the number of roots in the right half plane (RHP) can change only when some zeros appear and cross the imaginary axis. Therefore, a useful concept is the *frequency crossing set* Ω defined as the set of all real positive ω for which there exist at least a pair (k_p, k_i) such that

$$H(j\omega; k_p, k_i, \tau) := 1 + \frac{P(j\omega)}{Q(j\omega)}\left(k_p - j\frac{k_i}{\omega}\right)e^{-j\omega\tau} = 0 \quad (4)$$

We only need to consider positive frequencies ω, that is $\Omega \subset (0, \infty)$ since, obviously,

$$H(j\omega; k_p, k_i, \tau) = 0 \quad \Leftrightarrow \quad \overline{H(j\omega; k_p, k_i, \tau)} = 0.$$

Proposition 1 ([22]): For a given $\tau \in \mathbb{R}_+^*$ and $\omega \in \Omega$ the corresponding crossing point (k_p, k_i) is given by:

$$k_p = -\Re\left(\frac{Q(j\omega)}{P(j\omega)}e^{j\omega\tau}\right), \qquad (5)$$

$$k_i = \omega \cdot \Im\left(\frac{Q(j\omega)}{P(j\omega)}e^{j\omega\tau}\right). \qquad (6)$$

It is easy to see that $\forall\, \omega \in \Omega$ we have $P(j\omega) \neq 0$. Otherwise, $Q(j\omega) = 0$, that contradicts the *Assumption 1*.

Proposition 2 ([22]): Let k_p^* and $k_i^* > 0$ be given. Let $\Omega_{k_p^*, k_i^*}$ denotes the set of all frequencies $\omega > 0$ satisfying equation (4) for at least one pairs of (k_p, k_i) in the rectangle $|k_p| \leq k_p^*$, $|k_i| \leq k_i^*$. Then $\Omega_{k_p^*, k_i^*}$ consists of a finite number of intervals of finite length. Precisely, $\omega \in \Omega_{k_p^*, k_i^*}$ if and only if

$$\left|\frac{Q(j\omega)}{P(j\omega)}\right|^2 \leq \left(k_p^*\right)^2 + \frac{\left(k_i^*\right)^2}{\omega^2} \qquad (7)$$

Then, when ω varies within some interval Ω_l satisfying the inequality (7), (5)-(6) define a continuous curve. Denote this curve by $\mathcal{T}_l$ and consider the following decompositions:

$$R_0 + jI_0 = j\left.\frac{\partial H(s, k_p, k_i, \tau)}{\partial s}\right|_{s=j\omega},$$

$$R_1 + jI_1 = -\left.\frac{\partial H(s, k_p, k_i, \tau)}{\partial k_i}\right|_{s=j\omega},$$

$$R_2 + jI_2 = -\left.\frac{\partial H(s, k_p, k_i, \tau)}{\partial k_p}\right|_{s=j\omega}.$$

The implicit function theorem indicates that the tangent of $\mathcal{T}_l$ can be expressed as follows:

$$\begin{pmatrix} \dfrac{dk_p}{d\omega} \\ \dfrac{dk_i}{d\omega} \end{pmatrix} = \begin{pmatrix} R_2 & R_1 \\ I_2 & I_1 \end{pmatrix}^{-1} \begin{pmatrix} R_0 \\ I_0 \end{pmatrix} = \frac{1}{R_1 I_2 - R_2 I_1}\begin{pmatrix} R_1 I_0 - R_0 I_1 \\ R_0 I_2 - R_2 I_0 \end{pmatrix} \quad (8)$$

provided that

$$R_1 I_2 - R_2 I_1 \neq 0. \qquad (9)$$

In order to derive the stability region of the system given by (3), [22] characterized the smoothness of the crossing curves and the corresponding direction of crossing.

Proposition 3: The curve $\mathcal{T}_l$ is smooth every where except possibly at the point corresponding to $s = j\omega$ such that $s = j\omega$ is a multiple solution of (3).

Proposition 4: Assume $\omega \in \Omega_l$, k_p, k_i satisfy (5) and (6) respectively, and ω is a simple solution of (4) and

$$H(j\omega', k_p, k_i, \tau) \neq 0, \qquad \forall \omega' \neq \omega$$

(i.e. (k_p, k_i) is not an intersection point of two curves or different section of a single curve). Then, as (k_p, k_i) moves from the region on the right to the region on the left of the corresponding crossing curve, a pair of solution of (3) crosses the imaginary axis to the right (through $s = \pm j\omega$) if

$$R_1 I_2 - R_2 I_1 > 0.$$

The crossing is to the left if the inequality is reversed.

III. Main Result: Fragility of PI Controllers

Consider now the *PI fragility problem*, that is the problem of computing the maximum controller parameters deviation without losing the closed-loop stability – given the pair of parameters (k_p^*, k_i^*) such that the roots of the equation:

$$Q(s) + P(s)\Big(k_p^* + \frac{k_i^*}{s}\Big)e^{-s\tau} = 0,$$

are located in $\mathbb{C}_-$ (that is the closed-loop system is asymptotically stable), find the maximum parameter deviation $d \in \mathbb{R}_+$ such that the roots of (3) stay located in $\mathbb{C}_-$ for all controllers (k_p, k_i) satisfying:

$$\sqrt{(k_p - k_p^*)^2 + (k_i - k_i^*)^2} \leq d.$$

This problem can be more generally reformulated as: *find the maximum parameter deviation* d *such that the number of unstable roots of (3) remains unchanged.*

First, let us introduce some notation:

$$\mathcal{T} = \bigcup_{l=1}^{N} \mathcal{T}_l, \quad \mathcal{T}_l = \big\{(k_p, k_i) \big| \omega \in \Omega_l\big\}$$

$$\overrightarrow{k(\omega)} = (k_p(\omega), k_i(\omega))^T, \quad \overrightarrow{k^*} = \big(k_p^*, k_i^*\big)^T$$

Let us also denote $d_{\mathcal{T}} = \min_{l\in\{1,\ldots,N\}} d_l$, where

$$d_l = \min\Big\{\sqrt{(k_p - k_p^*)^2 + (k_i - k_i^*)^2} \ \mid (k_p, k_i) \in \mathcal{T}_l\Big\}$$

With the notation and the results above, we have:

Proposition 5: The maximum parameter deviation from (k_p^*, k_i^*), without changing the number of unstable roots of the closed-loop equation (3) can be expressed as:

$$d = \min\left\{|k_i^*|, \min_{\omega\in\Omega_f}\left\{\left\|\overrightarrow{k(\omega)} - \overrightarrow{k^*}\right\|\right\}\right\}, \qquad (10)$$

where Ω_f is the set of roots of the function $f : \mathbb{R}_+ \mapsto \mathbb{R}$,

$$f(\omega) \triangleq \left(\overrightarrow{k(\omega)} - \overrightarrow{k^*}\right) \cdot \frac{d\overrightarrow{k(\omega)}}{d\omega}, \qquad (11)$$

where " **.**" means the dot product.

Proof: We consider that the pair (k_p^*, k_i^*) belongs to a region generated by the crossing curves. Since the number of unstable roots changes only when (k_p, k_i) get out of this region, our objective is to compute the distance between (k_p^*, k_i^*) and the boundary of the region. Furthermore, the boundary of such a region consists of "pieces" of crossing curves and possibly one segment of the k_p axis. In order to compute the distance between (k_p^*, k_i^*) and a crossing curve we only need to identify the points where the vector $(k_p - k_p^*, k_i - k_i^*)$ and the tangent to the curve are orthogonal. In other words we have to find the solutions of

$$f(\omega) = 0,$$

where f is defined by (11). Taking into account the relation (8) we may write (11) as

$$f(\cdot) = \big(k_p - k_p^*\big)\left(R_1 I_0 - R_0 I_1\right) + \left(k_i - k_i^*\right)\left(R_0 I_2 - R_2 I_0\right)$$

It is noteworthy that $f(\omega)$ is a polynomial function and, therefore, it will have a *finite* number of roots. Let us consider $\{\omega_1, \ldots, \omega_M\}$ the set of all the roots of $f(\omega)$ when we take into account all the pieces of crossing curves belonging to the region around (k_p^*, k_i^*). Since the distance from (k_p^*, k_i^*) to the $k_p(\omega)$ axis is given by $|k_i^*|$, one obtains:

$$d = \min\left\{|k_i^*|, \min_{h=\{1,\ldots,M\}}\left\{\left\|\overrightarrow{k(\omega_h)} - \overrightarrow{k^*}\right\|\right\}\right\},$$

that is just another way to express (10). ■

The explicit computation of the maximum parameter deviation d can be summarized by the following algorithm:

- Step 1: First, compute the "degenerate" points of each curve $\mathcal{T}_l$ (i.e. the roots of $R_1 I_2 - R_2 I_1 = 0$ and the multiple solutions of (3)).
- Step 2: Second, compute the set Ω_f defined by *Proposition* 5 (i.e. the roots of equation $f(\omega) = 0$, where f is given by (11)).
- Step 3: Finally, the corresponding maximum parameter deviation d_l is defined by (10).

Remark 1 (On the gains' optimization): It is worth mentioning that the geometric argument above can be easily used for solving other *robustness problems*. Thus, for instance, if one of the controller's parameters is fixed (prescribed), we can also explicitly compute the *maximum* interval guaranteeing closed-loop stability with respect to the other parameter. In particular if T_i ("integral") is fixed, we can derive the corresponding stabilizing *maximum gain interval*. This gives a different insight to the results proposed by [31], [26] by using the small-gain theorem (see, for instance, the illustrative examples below).

IV. Illustrative Examples

Example 1 (Chemical Process): Consider the problem of controlling a continuous stirred tank reactor (CSTR) as in Fig.1 with the numerical values taken from [12] (see, e.g., [14], [29] for more details on CSTR). The goal is to control the reactor composition by manipulating the cool rate through the control signal u. Without getting into details, the transfer function of the system has the form:

$$H_{yu}(s) = -\frac{1.308}{(13.515s + 1)(6.241s + 1)}e^{-4.896s}. \qquad (12)$$

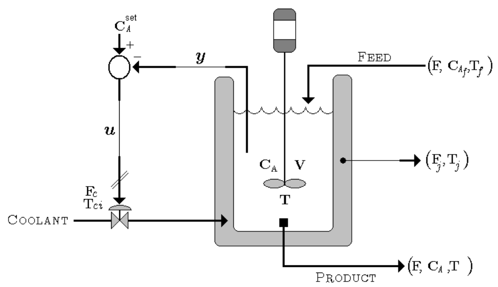

Fig. 1. A CSTR control system

The use of a PI-controller leads to $H(s; k_p, k_i) =:$

$$(13.515s+1)(6.241s+1)-1.308\left(k_p+\frac{k_i}{s}\right)e^{-4.896s}. \quad (13)$$

The system (13) has one stability region plotted in Fig.2.

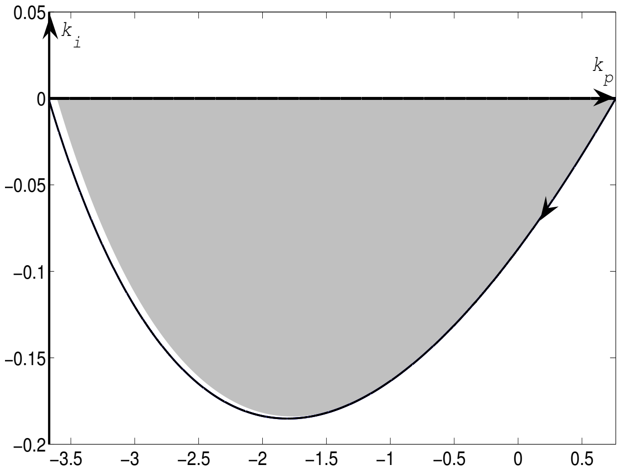

Fig. 2. The boundary of the stability region in the (k_p, k_i) parameters space for the system (12)

Next, we will study the fragility of PI-setting for some of the PI controllers proposed in the literature:

- Huang-Chou-Wuang[12]: $(k_p^* = -1.6881, k_i^* = -0.0732)$;
- Hwang[13]: $(k_p^* = -1.2173, k_i^* = -0.0529)$;
- Chao-Lin-Guu-Chang[3]: $(k_p^* = -1.1294, k_i^* = -0.0387)$;
- Ziegler-Nichols[33]: $(k_p^* = -1.4702, k_i^* = -0.0601)$.

By applying *Proposition 5*, the derived results are summarized in Table I and illustrated in the Fig.3.

	ω	$d_{\mathcal{T}}$	$\min\{d_{\mathcal{T}}, k_i^*\}$
Huang-Chou-Wang	0.1387	0.1114	0.0732
Hwang	0.1225	0.1202	0.0529
Chao-Lin-Guu-Chang	0.1194	0.1308	0.0387
Ziegler-Nichols	0.1323	0.1210	0.0601
Optimal Non-Fragile	0.1405	0.0925...	0.0925...

TABLE I

PI FRAGILITY COMPARISON FOR THE SYSTEM (12)

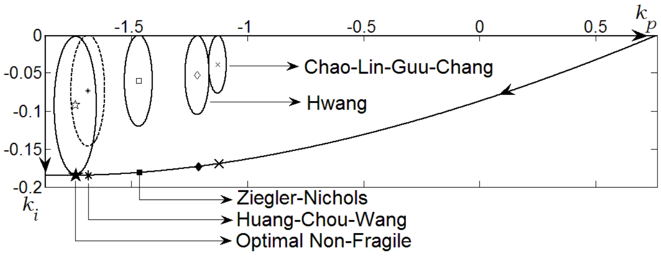

Fig. 3. The maximum parameter deviation without losing stability for the system (12), where the Optimal Non-Fragile controller is given by $k_p^* = -1.7420542840243\ldots$ and $k_i^* = -0.09250851510052\ldots$

Example 2 (A TCP/AQM network model): Consider the fluid-flow model introduced by [11] for describing the behavior of TCP/AQM networks and subject to PI controllers. As mentioned by [18], the stability of the linearized closed-loop system reduces to the root location of $H(s, k_p, k_i, \tau) :=$

$$s^2+\frac{1}{\tau}\left(1+\frac{n}{\tau c}\right)s+\frac{2n}{\tau^3 c}+\left[\frac{n}{\tau^2 c}s+\frac{c^2}{2n}\left(k_p+\frac{k_i}{s}\right)\right]e^{-\tau s}=0 \quad (14)$$

Here, n denotes the load factor (number of TCP sessions), τ the round-trip time (seconds) and c the link capacity (packets/sec). The crossing curves are given by:

$$k_p = \frac{2n}{c^2}\left[\left(\omega^2-\frac{2n}{\tau^3 c}\right)\cos(\omega\tau)+\frac{\omega}{\tau}\left(1+\frac{n}{\tau c}\right)\sin(\omega\tau)\right]$$

$$k_i = \frac{2n\omega}{c^2}\left[\frac{\omega}{\tau}\left(1+\frac{n}{\tau c}\right)\cos(\omega\tau)+\left(\frac{2n}{\tau^3 c}-\omega^2\right)\sin(\omega\tau) + \frac{n\omega}{\tau^2 c}\right]$$

Considering the same network parameters as in [11], [18] ($n = 60$, $c = 3750$, $\tau = 0.246$) and applying *Proposition 4* we get that all the crossing directions are towards instability. Furthemore, we have *only one stability region*. Consider now some of the controllers proposed in the literature:

- Melchor-Niculescu[18]: $(k_p^* = 9.1044 \times 10^{-5}, k_i^* = 6.8 \times 10^{-5})$;
- Hollot-Misra-Towsley-Gong[11]: $(k_p^* = 1.8485 \times 10^{-5}, k_i^* = 9.7749 \times 10^{-6})$;
- Üstebay-Özbay[30]: $(k_p^* = 3.5252 \times 10^{-5}, k_i^* = 8.9564 \times 10^{-6})$;
- Ziegler-Nichols[33]: $(k_p^* = 7.4401 \times 10^{-5}, k_i^* = 5.7057 \times 10^{-5})$;
- Huang-Chou-Wang[12]: $(k_p^* = 10.0011 \times 10^{-5}, k_i^* = 6.4880 \times 10^{-5})$.

The results are briefly outlined in the table II and illustrated in Fig.4.

	ω	$d_{\mathcal{T}}$ $[\times 10^{-5}]$	$\min\{d_{\mathcal{T}_l}, \lvert k_i^*\rvert\}$ $[\times 10^{-5}]$
Melchor and Niculescu	$\omega_1 = 1.76$ $\omega_2 = 2.75$ $\omega_3 = 3.49$	$d_{\mathcal{T}_1} = 6.74$ $d_{\mathcal{T}_2} = 8.78$ $d_{\mathcal{T}_3} = 6.82$	6.7410
Hollot-Misra and Towsley-Gong	$\omega_1 = 0.72$ $\omega_2 = 3.00$ $\omega_3 = 3.69$	$d_{\mathcal{T}_1} = 3.00$ $d_{\mathcal{T}2} = 17.0$ $d_{\mathcal{T}2} = 15.6$	0.9774
Üstebay and Özbay	$\omega_1 = 0.81$ $\omega_2 = 2.93$ $\omega_3 = 3.72$	$d_{\mathcal{T}_1} = 4.56$ $d_{\mathcal{T}_2} = 16.2$ $d_{\mathcal{T}_3} = 14.0$	0.8956
Ziegler and Nichols	$\omega_1 = 1.55$ $\omega_2 = 2.85$ $\omega_3 = 3.52$	$d_{\mathcal{T}_1} = 5.89$ $d_{\mathcal{T}_2} = 10.2$ $d_{\mathcal{T}_3} = 8.77$	5.7057
Huang Chou and Wang	$\omega_1 = 1.79$ $\omega_2 = 2.68$ $\omega_3 = 3.53$	$d_{\mathcal{T}_1} = 7.65$ $d_{\mathcal{T}_2} = 9.07$ $d_{\mathcal{T}_3} = 6.10$	6.1094

TABLE II

PI-FRAGILITY COMPARISON FOR THE CHARACTERISTIC EQUATION (14)

Remark 2: As mentioned in the previous chapter, it is also possible to solve the following problem – given a fixed integral (gain) parameter $T_i = \frac{k_p}{k_i}$, find the *optimal* interval for the gain (integral) parameter $k_p = k$, such that, the

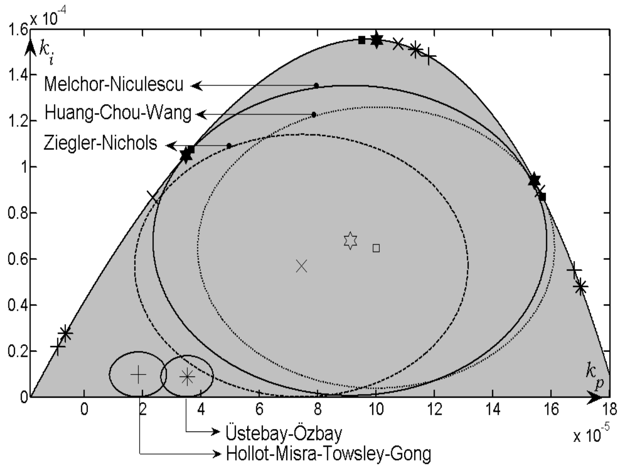

Fig. 4. Fragility comparison of the PI-controllers for the system

resulting closed-loop system is stable for all gain (integral) parameters In this case, it is sufficient to find the "mid-point" of the maximal interval which belong to the stability region. Reconsider the previous controllers:

- "optimal" gain (Hollot-Misra-Towsley-Gong): $k = 7.91 \times 10^{-5}$;
- "optimal" gain (Üstebay-Özbay): $k = 8.56 \times 10^{-5}$;
- "optimal" gain (Melchor-Niculescu): $k = 7.34 \times 10^{-5}$

It is easy to see that the controller proposed by Üstebay-Özbay is "closer" to the "non-fragile" one than Hollot-Misra-Towsley-Gong. The above results are also depicted in Fig.5

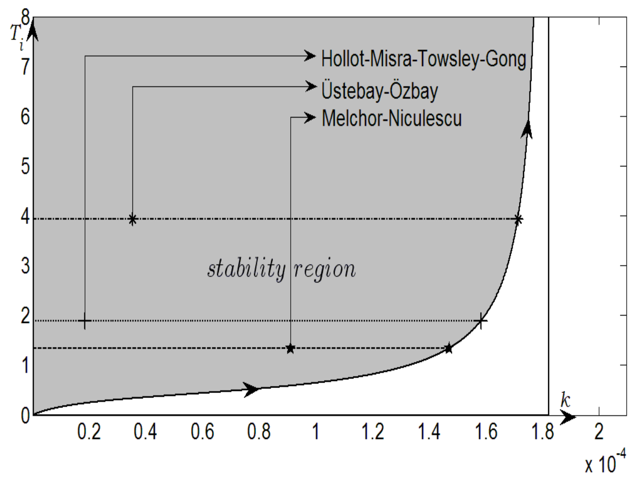

Fig. 5. Gain fragility comparison of the PI-controllers for the system (14)

Example 3 (Unstable, non-minimum phase): Consider a second-order, non-minimum-phase and unstable open-loop system, described by the transfer function:

$$H_{yu} = \frac{(s-2)e^{-2s}}{s^2 - 1/2s + 13/4}, \qquad (15)$$

leading to the closed-loop equation:

$$2 - \frac{1}{2}s + \frac{13}{4} + (s-2)\Big(k_p + \frac{k_i}{s}\Big)e^{-2s} = 0 \qquad (16)$$

Fig.6 depicts the stability region and the "optimal" non-fragile controller is given by $(k_p^*, k_i^*) = (-0.4959, -0.3559\ldots)$ (see also the Table III).

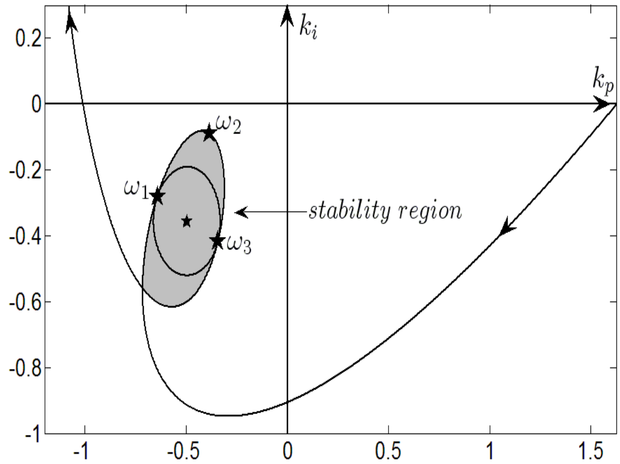

Fig. 6. The boundary of the stability region in the (k_p, k_i) parameters space, together with the maximum parameter deviation without losing stability for the system (15)

Frequency	$d_{\mathcal{T}_I}$	$\lvert k_i^* \rvert$	$\min\{d_{\mathcal{T}}, \lvert k_i^* \rvert\}$
$\omega_1 = 1.3294$	0.1649067	0.355948	0.1649067
$\omega_2 = 1.6313$	0.2888059		
$\omega_3 = 1.9530$	0.1649067		

TABLE III

PARAMETER DEVIATION RESULTS WITHOUT LOSING THE STABILITY

Example 4 (Fourth-order process): Consider a fourth-order, non-minimum-phase and unstable open-loop system, with the transfer function:

$$H_{yu}(s) = \frac{(-1.3s+3)e^{-2.8s}}{0.2s^4 - 0.08s^3 + 1.345s^2 - 0.4s + 1.725}. \qquad (17)$$

Similarly to the previous cases, the problems reduces to the analysis of equation:

$$0.2s^4 - 0.08s^3 + 1.345s^2 - 0.4s + 1.725 + (-1.3s+3)\left(k_p + \frac{k_i}{s}\right)e^{-2.8s} = 0 \qquad (18)$$

The "optimal" non-fragile PI-controller for the system (17) is given by $(k_p^*, k_i^*) = (0.1149\ldots, 0.0778\ldots)$ (see also Table IV and Fig.7):

Frequency	$d_{\mathcal{T}_I}$	$\lvert k_i^* \rvert$	$\min\{d_{\mathcal{T}}, \lvert k_i^* \rvert\}$
$\omega_1 = 1.2311$	0.0313616	0.077849	0.0311658
$\omega_2 = 1.2422$	0.0313627		
$\omega_3 = 1.3232$	0.0311658		
$\omega_4 = 1.5556$	0.0400741		
$\omega_5 = 1.7025$	0.0311658		

TABLE IV

PARAMETERS DEVIATION RESULTS WITHOUT LOSING THE STABILITY

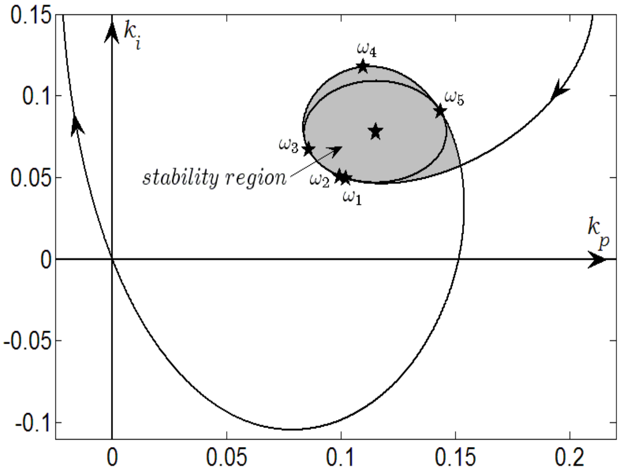

Fig. 7. The stability crossing curves for the dynamic system (17), the boundary of the stability region (shadowed region) in the (k_p, k_i) parameters space and the maximum parameter deviation without losing stability

V. Concluding Remarks

In this paper, we have developed a simple geometrical method for computing the fragility of PI-controllers for a class of strictly proper SISO systems with I/O delays. To prove the efficiency of the method, several illustrative examples have been considered. It is important to note that such an idea can be easily extended to *proper* SISO systems with I/O delays as well as to the case of PD controllers.

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